1.- Use the definition of the limit of a sequence to show that:

$$\lim \left(\frac{n}{n^2+1} \right) = 0$$

2 - Establish the convergence or the divergence of the sequence $\{S_n\}$, where:

$$S_n := \frac{1}{n+1} + \frac{1}{n+2} + \cdots \frac{1}{2n}, \quad \text{for } n = 1, 2, 3, \ldots$$

3.- Let $\{S_n\}$ be a sequence of real numbers. Suppose that every subsequence of $\{S_n\}$ has a subsequence that converges to 0. Show that:

$$\lim S_n = 0$$

4.- Let $\{S_n\}$ be a bounded sequence of real numbers. Then $\{S_n\}$ is convergent if and only if $\liminf\limits_{n\to\infty} S_n = \limsup\limits_{n\to\infty} S_n$ in which case $\lim\limits_{n\to\infty} S_n$ is the common value.