Homework 9.

1.- Let $I = [a, b]$ and let $f : I \rightarrow \mathbb{R}$ be a bounded function and α be a monotonically increasing function on $[a, b]$. Show that if $\{P_n\}_{n=1}^{\infty}$ is a sequence of partitions of I such that:

$$\lim_{n \to \infty} (U(P_n, f, \alpha) - L(P_n, f, \alpha)) = 0$$

then $f \in \mathcal{R}(\alpha)$ and:

$$\lim_{n \to \infty} L(P_n, f, \alpha) = \int_a^b f \, d\alpha = \lim_{n \to \infty} U(P_n, f, \alpha).$$

2.- Use previous problem to show that:

$$\int_0^1 x = \frac{1}{2}$$