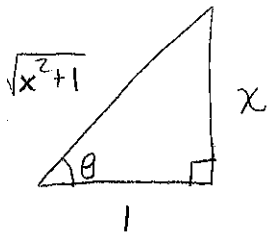
MA161 - Quiz 8 - Fall 2016

Name: _____

Directions: Please show all your work leading to your answers. Having some correct work with an incorrect answer will earn you partial credit.

1. Compute $\sin(\tan^{-1} x)$. (Your answer should not contain trigonometric functions) (4 points)

   Solution: Let $\theta = \tan^{-1} x$, so that $\tan\theta = x = \dfrac{x}{1}$. We can then draw the following right triangle.



   The triangle has x on the opposite side and 1 on the adjacent side since these values mean $\tan\theta = x$, then the hypotenuse is filled in from the Pythagorean theorem. Then $\sin(\tan^{-1} x) = \sin\theta = \dfrac{x}{\sqrt{x^2+1}}$.

   (This picture technically doesn't make sense if $x < 0$, but the solution still works. If $x < 0$ we should replace x with $-x$ and θ with $-\theta$ - then everything works the same.)

2. For the following functions, compute the derivative with respect to x. (2 points each)

   (a) $f(x) = \sec x$ **Solution:** $f'(x) = \sec x \tan x$
   (b) $f(x) = \cot x$ **Solution:** $f'(x) = -\csc^2 x$
   (c) $f(x) = \sin x$ **Solution:** $f'(x) = \cos x$
   (d) $f(x) = \tan x$ **Solution:** $f'(x) = \sec^2 x$
   (e) $f(x) = \cos x$ **Solution:** $f'(x) = -\sin x$
   (f) $f(x) = \csc x$ **Solution:** $f'(x) = -\csc x \cot x$

3. Find the equation of the tangent line to the graph of $g(x) = \dfrac{x}{\tan x}$ at the point $\left(\dfrac{\pi}{4}, g\left(\dfrac{\pi}{4}\right)\right)$. (Your answer should not contain trigonometric functions.) (4 points)

   Solution: The derivative of g is

$$g'(x) = \frac{\tan x - x\sec^2 x}{\tan^2 x}.$$

   Since $\tan(\pi/4) = 1$ and $\sec(\pi/4) = \sqrt{2}$, $g'(\pi/4) = 1 - \dfrac{\pi}{2}$. Since $g(\pi/4) = \pi/4$, the equation of the tangent line is

$$y - \frac{\pi}{4} = \left(1 - \frac{\pi}{2}\right)\left(x - \frac{\pi}{4}\right)$$