MA26100 – Exam II – Summer 2016

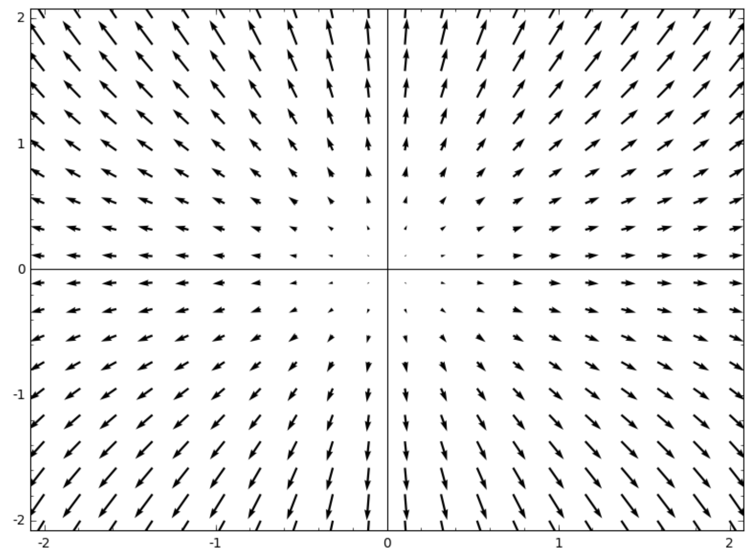
STUDENT NAME ________________________________

STUDENT ID ________________________________

INSTRUCTIONS

1. You'll have 1 hour to complete this exam.
2. Fill in your name and Student ID
3. Put your answers in the box provided on each page, or circle your answers. If it's not clear what your answer is, you'll receive little to no credit.
4. There are 10 questions: 2 Multiple Choice (CIRCLE YOUR ANSWER!), and 8 Short Answer.
5. Point values are displayed next to the question number.
6. Show all work involved. Little to no work will receive little to no credit.
7. Keep your eyes on your own exam.
8. No books, calculators, phones, laptops, etc. are allowed out during the exam.

1. (6 pts) Which of the following vector fields could match the plot given below?



A. $x\,\mathbf{i} + \frac{1}{y}\,\mathbf{j}$

B. $e^x\,\mathbf{i} + y^2\,\mathbf{j}$

C. $-x\,\mathbf{i} + \sqrt{y}\,\mathbf{j}$

D. $x^3\,\mathbf{i} + y^2\,\mathbf{j}$

E. $\sin(x)\,\mathbf{i} + y\,\mathbf{j}$

2. (6 pts) Suppose $f(x, y) = 1/2\left(\sin^2(x) + \sin^2(y) - \cos^2(x) - \cos^2(y)\right)$. For this function, the function $D(x, y) = f_{xx}(x, y)f_{yy}(x, y) - (f_{xy}(x, y))^2$ is given by

$$D(x, y) = 4\cos(2x)\cos(2y)$$

Which of the following statements are true?

I. $(\pi, \pi/2)$ is a saddle point of f

II. (π, π) is a local minimum of f

III. $(\pi/8, \pi/8)$ is a local minimum of f

A. Only I

B. Only II

C. Only III

D. Only I and II

E. I, II, and III

3. (11 pts) Let E be the region bounded by the surfaces

$$z = 0$$
$$z = \sqrt{4 - x^2 - y^2}$$
$$x = y$$
$$y = 0$$

(a) (5 pts) Fill in the appropriate information so the following triple integral gives the volume of E

$$\int \int \int \underline{\qquad\qquad} \, dz\, dx\, dy$$

(b) (6 pts) Set up and evaluate using spherical coordinates the triple integral that gives the volume of E.

4. (10 pts) Find the minimum distance between the point $P(1, 0, 2)$ and the plane

$$x + y + z = 1$$

5. (12 pts) Find the absolute maximum and mininmum values of

$$f(x,y) = \frac{xy}{1 + x^2 + y^2}$$

On the set $E = \{(x,y) \mid -1 \leq x \leq 1,\ -1 \leq y \leq 1\}$

Maximum:

Minimum:

6. (10 pts) Rewrite the following as one double integral by changing to polar coordinates:

$$\int_{\frac{1}{\sqrt{2}}}^{1} \int_{y}^{\sqrt{2-y^2}} f(x,y)\,dx\,dy + \int_{-\frac{1}{\sqrt{2}}}^{\frac{1}{\sqrt{2}}} \int_{\sqrt{1-y^2}}^{\sqrt{2-y^2}} f(x,y)\,dx\,dy$$

$$+ \int_{-1}^{-\frac{1}{\sqrt{2}}} \int_{-y}^{\sqrt{2-y^2}} f(x,y)\,dx\,dy$$

7. (12 pts) You have a plate that is described by the inequality below:

$$x^2 + y^2 \leq 1$$

Its density is given by $\rho(x, y) = x^2 - x + 1 + y^2$. ($\rho$ has units $\frac{\text{g}}{\text{m}^2}$)

(a) (4 pts) Find the mass of the plate.

(b) (6 pts) Find M_x (the moment about the $x-$axis) for this plate.

8. (11 pts) The temperature at any point (x, y, z) in this room is given by

$$T(x, y, z) = 200e^{-x^2-3y^2-9z^2}$$

In what direction does the temperature increase the fastest at $P(2, -1, 2)$? (Your answer should be a vector, but it need not be a unit vector).

9. (11 pts) Find the surface area of the part of the paraboloid $z = 4 - x^2 - y^2$ that lies above the $xy-$ plane.

10. (11 pts) Evaluate

$$\oint_C x^2 + y\, ds$$

where C is the curve that goes from the origin to $(1,0)$ along a straight line, then from $(1,0)$ to $(0,1)$ along the unit circle, and then returns from $(0,1)$ to $(0,0)$ along a straight line.

(You might find the identity $\cos^2(x) = \frac{1+\cos(2x)}{2}$ handy.)