**1.** (9 points) Find an equation of the plane that contains the line $\mathbf{r}(t) = \langle 1, 0, 1\rangle + t\langle -1, 1, 2\rangle$ and the origin.

$\langle 1,0,1\rangle - \langle 0,0,0\rangle = \langle 1,0,1\rangle$ is a vector parallel to the plane and $\langle -1,1,2\rangle$ is another such vector, so

$$\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & 1 \\ -1 & 1 & 2 \end{vmatrix} = \begin{vmatrix} 0 & 1 \\ 1 & 2 \end{vmatrix}\hat{i} - \begin{vmatrix} 1 & 1 \\ -1 & 2 \end{vmatrix}\hat{j} + \begin{vmatrix} 1 & 0 \\ -1 & 1 \end{vmatrix}\hat{k} = -\hat{i} - 3\hat{j} + \hat{k}.$$

Since the plane contains $(0,0,0)$, an equation is C.

A. $-x + 4y + z = 0$

B. $4x + 2x - 4z = 0$

C. $-x - 3y + z = 0$

D. $x + y - z = 0$

E. $2x + 3y - 2z = 0$

**2.** (9 points) Find the curvature of the helix

$$\mathbf{r}(t) = \langle \cos t, \sin t, t\rangle.$$

Recall: $\kappa = \dfrac{|\mathbf{T}'|}{|\mathbf{r}'|}$

$\vec{r}'(t) = \langle -\sin t, \cos t, 1\rangle$, $|\vec{r}'(t)| = \sqrt{2}$.

$\vec{T}(t) = \frac{1}{\sqrt{2}}\langle -\sin t, \cos t, 1\rangle$

$\vec{T}'(t) = \frac{1}{\sqrt{2}}\langle -\cos t, -\sin t, 0\rangle$

$|\vec{T}'(t)| = \frac{1}{\sqrt{2}} \Rightarrow \kappa = \frac{1/\sqrt{2}}{\sqrt{2}} = \frac{1}{2}.$

A. $2\sqrt{2}$

B. $1/2$

C. $\sqrt{2}$

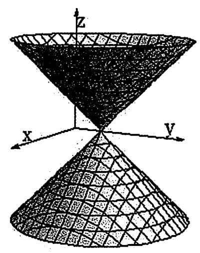
D. $\sqrt{2}/2$

E. $1$

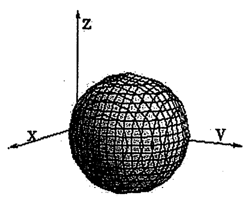
**3.** (9 points) Which of the graphs below is the graph of the given equation?

$$x^2 + y^2 - z^2 - 2y = 0$$

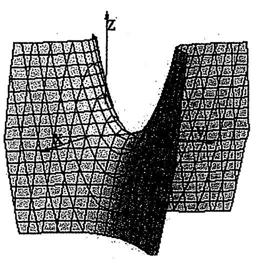
$x^2+(y-1)^2-z^2=1$ is the standard equation of a hyperboloid of one sheet opening around the z-axis that has been shifted 1 unit in the positive y-direction.



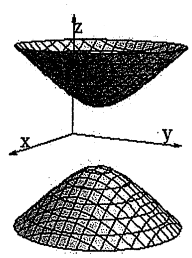

A.



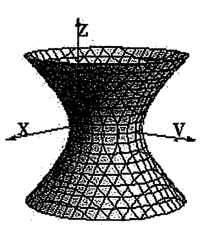

B.




C.




D.




E.

**4.** (9 points) Use the linearization of $f(x,y) = \sqrt{x^2+y^2}$ at (3,4) to approximate the number $\sqrt{(3.1)^2+(3.8)^2}$.

$f_x = \frac{x}{\sqrt{x^2+y^2}}$ , $f_y = \frac{y}{\sqrt{x^2+y^2}}$

$L(x,y) = 5 + \frac{3}{5}(x-3) + \frac{4}{5}(y-4)$

$\sqrt{(3.1)^2+(3.8)^2} = f(3.1, 3.8) \approx L(3.1, 3.8) = 5 + \frac{3}{5}(0.1) + \frac{4}{5}(-0.2)$

$= 5 + \frac{6}{100} - \frac{16}{100}$

$= 4.90$.

A. 4.84
B. 4.86
C. 4.88
D. 4.90
E. 4.92

**5.** (8 points) Find all possible values of $a$ so that the angle between the vectors

$$\mathbf{a} = \left\langle \frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}} \right\rangle, \mathbf{b} = \langle 0, a, 1 \rangle$$

is $\pi/3$.

$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta$

$\Rightarrow \frac{1}{\sqrt{2}} = 1 \cdot \sqrt{a^2+1} \cos\frac{\pi}{3}$

$\frac{1}{\sqrt{2}} = \sqrt{a^2+1} \cdot \frac{1}{2}$

$\sqrt{2} = \sqrt{a^2+1}$

$2 = a^2+1$

$1 = a^2$

$\pm 1 = a$

$\boxed{1, -1}$

**6.** (10 points) Find parametric equations for the tangent line to the curve

$$\mathbf{r}(t) = \langle \sin(2\pi t), t^2 + 2t, \arctan(t) \rangle$$

at the point $(0, 3, \pi/4)$.

Note that the unique $t$ with $\vec{r}(t) = (0, 3, \pi/4)$ is $t = 1$ (since the only $t$ with $\arctan t = \frac{\pi}{4}$ is 1).

$$\vec{r}\,'(t) = \langle 2\pi\cos(2\pi t), 2t+2, \frac{1}{1+t^2} \rangle$$

$$\vec{r}\,'(1) = \langle 2\pi, 4, \frac{1}{2} \rangle.$$

Vector equation: $\vec{L}(t) = \langle 0, 3, \frac{\pi}{4} \rangle + t\langle 2\pi, 4, \frac{1}{2} \rangle$

$$\boxed{x = 2\pi t,\ y = 4t + 3,\ z = \frac{t}{2} + \frac{\pi}{4}.}$$

**7.** (12 points) A particle is moving in space with acceleration

$$\mathbf{a}(t) = \pi^2 \cos(\pi t)\hat{\mathbf{i}} + \frac{1}{(t+1)^2}\hat{\mathbf{j}} + e^{t/2}\hat{\mathbf{k}}.$$

Assume the particle is initially at rest (that is, it has no initial speed) and its initial position is the origin. What is the particle's position at $t = 1$?

$\vec{v}(0) = \vec{0}, \quad \vec{r}(0) = \vec{0}.$

$\vec{v}(t) = \pi\sin(\pi t)\hat{i} - \frac{1}{t+1}\hat{j} + 2e^{t/2}\hat{k} + \vec{v_0}$

Plug in 0: $\vec{0} = -\hat{j} + 2\hat{k} + \vec{v_0}$

$\Rightarrow \vec{v_0} = \hat{j} - 2\hat{k}.$

$\vec{v}(t) = \pi\sin(\pi t)\hat{i} + \left(1 - \frac{1}{t+1}\right)\hat{j} + (2e^{t/2} - 2)\hat{k}.$

$\vec{r}(t) = -\cos(\pi t)\hat{i} + (t - \ln|t+1|)\hat{j} + (4e^{t/2} - 2t)\hat{k} + \vec{r_0}$

Plug in 0: $\vec{0} = -\hat{i} + 4\hat{k} + \vec{r_0}$

$\Rightarrow \vec{r_0} = \hat{i} - 4\hat{k}.$

$\vec{r}(t) = (1 - \cos(\pi t))\hat{i} + (t - \ln|t+1|)\hat{j} + (4e^{t/2} - 2t - 4)\hat{k}$

$$\boxed{\vec{r}(1) = \langle 2, 1 - \ln 2, 4\sqrt{e} - 6 \rangle}$$

**8.** (8 points) Sketch the domain of

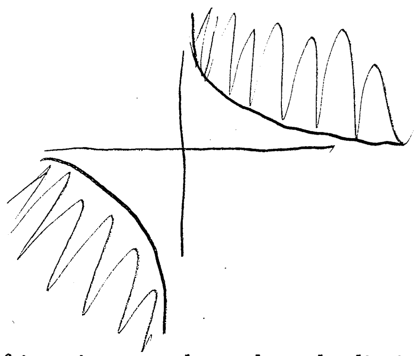
$$f(x, y) = \sqrt{xy - 1}.$$

**Circle your answer.**

$(x,y)$ is in the domain $\Leftrightarrow xy-1 \geq 0 \Leftrightarrow xy \geq 1.$

For $x>0$; $y \geq \frac{1}{x}$

For $x<0$; $y \leq \frac{1}{x}$



**9.** (8 points) Find the limit, if it exists, or show that the limit does not exist. If the limit does not exist, write DNE in the box.

$$\lim_{(x,y)\to(0,0)} \frac{x^2ye^y}{x^4+4y^2}$$

If $y=0$: $\lim_{x\to 0} \frac{0}{x^4} = 0.$

If $y=x^2$: $\lim_{x\to 0} \frac{x^4e^{x^2}}{x^4+4x^4} = \lim_{x\to 0} \frac{x^4e^{x^2}}{5x^4} = \lim_{x\to 0} \frac{e^{x^2}}{5} = \frac{1}{5}.$

$\boxed{\text{DNE}}$

**10.** (8 points) Let $f(x,y) = \cos(\pi x^2 - 3xy)$. Find an equation of the plane tangent to the graph of $f$ at the point $(1, \pi/4, \sqrt{2}/2)$.

$f_x(x,y) = -\sin(\pi x^2 - 3xy)\cdot(2\pi x - 3y) = (3y - 2\pi x)\sin(\pi x^2 - 3xy)$

$f_y(x,y) = -\sin(\pi x^2 - 3xy)(-3x) = 3x\sin(\pi x^2 - 3xy)$

$f_x(1, \frac{\pi}{4}) = (\frac{3\pi}{4} - 2\pi)\sin(\pi - \frac{3\pi}{4}) = -\frac{5\pi}{4}\sin(\frac{\pi}{4}) = -\frac{5\sqrt{2}\pi}{8}$

$f_y(1, \frac{\pi}{4}) = 3\sin(\frac{\pi}{4}) = \frac{3\sqrt{2}}{2}$.

$$\boxed{z - \frac{\sqrt{2}}{2} = -\frac{5\sqrt{2}\pi}{8}(x-1) + \frac{3\sqrt{2}}{2}(y - \frac{\pi}{4})}$$

**11.** (10 points) **Ohm's Law** for a simple electric circuit is

$$V = IR$$

where $V$ is the voltage (in volts), $I$ is the current (in amperes), and $R$ is the resistance (in ohms). In a simple circuit, suppose $R = 4\ \Omega$ ($\Omega$ is the symbol for ohms), $I = 5$ A, the voltage is decreasing at 1 V/s, and the resistance is increasing at 3 $\Omega$/s. At what rate is the current $I$ changing?

$V(I,R) = IR$.

Using the Chain Rule,

$$\frac{dV}{dt} = \frac{\partial V}{\partial I}\frac{dI}{dt} + \frac{\partial V}{\partial R}\frac{dR}{dt}$$

$$= R\frac{dI}{dt} + I\frac{dR}{dt}$$

$$\Rightarrow -1 = 4\cdot\frac{dI}{dt} + 5\cdot 3$$

$$-16 = 4\frac{dI}{dt}$$

$$-4 = \frac{dI}{dt}$$

$$\boxed{-4\ A/s}$$