Quiz #10

a)

$$x = 2y + z^2 \Rightarrow y = y\ ,\quad z = z$$

$$\boxed{\vec{r}(y,z) = (2y+z^2)\vec{i} + y\vec{j} + z\vec{k}}$$

b)

$$\left.\begin{aligned} \vec{r}_y &= 2\vec{i} + \vec{j} \\ \vec{r}_z &= 2z\vec{i} + \vec{k} \end{aligned}\right\} \Rightarrow \vec{r}_y \times \vec{r}_z = (2\vec{i} + \vec{j}) \times (2z\vec{i} + \vec{k}) =$$

$$= -2\vec{j} - 2z\vec{k} + \vec{i} \Rightarrow$$

The normal of the tangent plane at the origin is $\vec{n} = \vec{i} - 2\vec{j}$, hence the tangent plane to the surface S at the origin is

$$\vec{n} \cdot (\vec{r} - \vec{r}_0) = (\vec{i} - 2\vec{j}) \cdot \left[(x-0)\vec{i} + (y-0)\vec{j} + (z-0)\vec{k}\right] = 0$$

$$= x - 2y = 0 \Rightarrow \boxed{y = \tfrac{1}{2}x}$$

Tangent plane at origin to S

c)

$$|\vec{r}_y \times \vec{r}_z| = \sqrt{(-2)^2 + (-2z)^2 + 1} = \sqrt{5+4z^2}$$

$$\iint_D f(\vec{r}(y,z))\,|\vec{r}_y \times \vec{r}_z|\,dA = \iint_D z\sqrt{5+4z^2}\,dA$$

By Fubini's theorem

$$= \int_0^1\int_0^1 z\sqrt{5+4z^2}\,dy\,dz = \int_0^1 \left[yz\sqrt{5+4z^2}\right]_0^1 dz$$

$$= \int_0^1 z\sqrt{5+4z^2}\,dz = \frac{1}{8}\left[\frac{2}{3}(5+4z^2)^{3/2}\right]_0^1 =$$

$$= \frac{1}{12}\left(27 - 5\sqrt{5}\right)$$