

1)

Quiz #2

$$\int \vec{r}(t) dt = \int t^3 dt \vec{i} + \int \frac{2t}{t^2+1} dt \vec{j} + \int \cos(3t) dt \vec{k}$$

$$= \frac{t^4}{4} \vec{i} - \ln(t^2+1) \vec{j} + \frac{1}{3} \sin 3t \vec{k} + \underline{\underline{\vec{r}_0}}$$

(5)

$$2) L = \int_0^2 |\vec{r}'(t)| dt$$

$$\vec{r}'(t) = t^2 \vec{i} + 4 \vec{j} + 2\sqrt{2} t \vec{k}$$

$$|\vec{r}'(t)| = \sqrt{t^4 + 16 + 8t^2} = \sqrt{(t^2+4)^2} = |t^2+4|$$

$$= (t^2+4) \quad \text{since } 0 \leq t \leq 2$$

$$\Rightarrow L = \int_0^2 (t^2+4) dt = \left(\frac{t^3}{3} + 4t \right)_0^2 = \frac{8}{3} + 8 = \frac{32}{3}$$

(6)

Quiz #2

$$3) \quad \vec{r}(t) = e^{-t} \vec{i} + \sin t \vec{j} + t^2 \vec{k}$$

What is curvature $\kappa(t)$ at $t=0$?

(9)

$$\left. \begin{aligned} \vec{r}'(t) &= -e^{-t} \vec{i} + \cos t \vec{j} \\ \vec{r}''(t) &= e^{-t} \vec{i} - \sin t \vec{j} \end{aligned} \right\} \Rightarrow$$

$$\begin{aligned} \vec{r}'(t) \times \vec{r}''(t) &= (-e^{-t} \vec{i} + \cos t \vec{j}) \times (e^{-t} \vec{i} - \sin t \vec{j}) = \\ &= +e^{-t} \sin t \vec{k} - e^{-t} \cos t \vec{k} = \end{aligned}$$

$$\boxed{\vec{r}' \times \vec{r}'' = e^{-t} (\sin t - \cos t) \vec{k}} \Rightarrow$$

$$|\vec{r}' \times \vec{r}''| = \sqrt{(e^{-t} (\sin t - \cos t))^2} = |e^{-t} (\sin t - \cos t)|$$

$$|\vec{r}'(t)| = \sqrt{(-e^{-t})^2 + (\cos t)^2} = \sqrt{e^{-2t} + \cos^2 t}$$

$$\kappa(t) = \frac{|\vec{r}' \times \vec{r}''|}{|\vec{r}'|^3} = \frac{|e^{-t} (\sin t - \cos t)|}{(\sqrt{e^{-2t} + \cos^2 t})^3} \bigg|_{t=0} = \frac{-1}{2^{3/2}} = -\frac{\sqrt{2}}{4}$$