Quiz #4

1) $f(x,y) = 4x^2 + 10y^2$

$f_x = 8x$, $f_y = 20y$, $f_{xy} = 0$

$f_{xx} = 8$ $\quad f_{yy} = 20$

$\left.\begin{array}{l} f_x = 0 \Rightarrow x = 0 \\ f_y = 0 \Rightarrow y = 0 \end{array}\right] \Rightarrow f(0,0) = 0 \Longrightarrow$

$\boxed{\text{critical point } (0,0,0)}$

$D(0,0) = f_{xx}(0,0)\, f_{yy}(0,0) - f_{xy}(0,0) = (20)(8) > 0$

$f_{xx}(0,0) = 8 > 0 \Longrightarrow \boxed{(0,0,0) \text{ local Minimum}}$

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2) $\nabla f = \langle f_x, f_y \rangle = \langle 8x, 20y \rangle$

$\nabla g = \langle g_x, g_y \rangle = \langle 2x, 2y \rangle$

$\nabla f = \lambda \nabla g \Longrightarrow \langle 8x, 20y \rangle = \lambda \langle 2x, 2y \rangle$

$$\Longrightarrow \begin{cases} 8x = 2\lambda x & (1) \\ 20y = 2\lambda y & (2) \end{cases} \quad \text{and} \quad x^2+y^2 = 4 \quad (3)$$

$(1) \Rightarrow \quad 2x(4-\lambda) = 0 \quad \Rightarrow \quad x=0$ or $\lambda = 4$

Note $x=0$, $y=0$ contradicts (3)

$x=0$, $(3) \Rightarrow \left.\begin{matrix} y=\pm 2 \\ (2) \end{matrix}\right\} \Rightarrow \lambda = 10$

$\lambda = 4$, $(2) \Rightarrow \left.\begin{matrix} y=0 \\ (3) \end{matrix}\right\} \Rightarrow x = \pm 2$

Herce the possible extreme values of f happen at $(0, \pm 2)$, $(\pm 2, 0)$. So by checkig the values of f at the points:

$f(0,2) = 40$ $\qquad$ $f(2,0) = 16$

$f(0,-2) = 40$ $\qquad$ $f(-2,0) = 16$

$\boxed{\begin{array}{l} \text{Max of } f \text{ is } 40 \text{ at } (0,\pm 2) \\ \text{Min of } f \text{ is } 16 \text{ at } (\pm 2, 0) \end{array}}$

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3) From part 1 the minimum of f in D is $f(0,0) = 0$ (there is no local Max of f in D)

From part 2, the max f and min f on the boundary of D are $f(0,\pm 2) = 40$ and $f(\pm 2, 0) = 16$ respectively.

Hence the absolute max $f = f(0,\pm 2) = 40$ and absolute min $f = f(0,0) = 0$