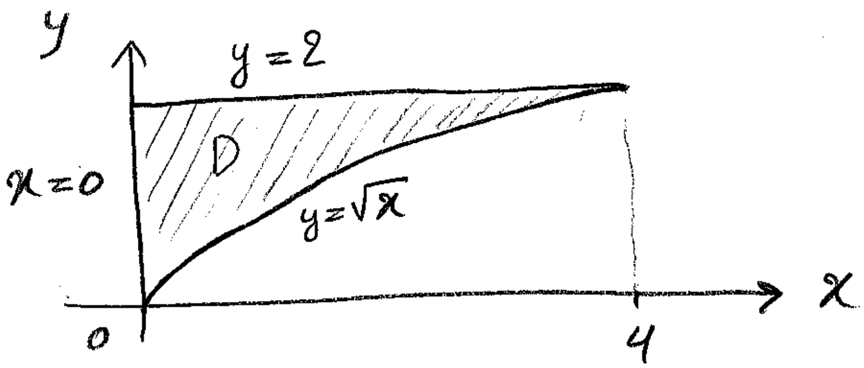
Quiz #5

a) $D = \{(x,y) \mid 0 \leq x \leq 4,\ \sqrt{x} \leq y \leq 2\}$



b) $y = \sqrt{x} \Rightarrow x = y^2 \Rightarrow$ using the sketch above

$$D = \left\{ (x,y) \,\middle|\, \begin{array}{l} 0 \leq x \leq 4 \\ \sqrt{x} \leq y \leq 2 \end{array} \right\} = \left\{ (x,y) \,\middle|\, \begin{array}{l} 0 \leq x \leq y^2 \\ 0 \leq y\ 2 \end{array} \right\}$$

Hence

$$\iint_D e^{y^3}\, dA = \int_0^4 \int_{\sqrt{x}}^2 e^{y^3}\, dy\, dx = \int_0^2 \int_0^{y^2} e^{y^3}\, dx\, dy =$$

$$= \int_0^2 \left[x e^{y^3} \right]_0^{y^2} dy = \int_0^2 y^2 e^{y^3} dy =$$

$$= \frac{1}{3} \left[e^{y^3} \right]_0^2 = \frac{1}{3} \left(e^8 - 1 \right)$$

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c) Area of $D = A(D) = 8 - \int_0^4 \sqrt{x}\, dx$

$$\int_0^4 \sqrt{x}\, dx = \left[\frac{2}{3} x^{3/2} \right]_0^4 = \frac{2}{3} (4)^{3/2} = \frac{16}{3}$$

$$A(D) = 8 - \frac{16}{3} = \frac{8}{3}$$

$$f_{ave} = \frac{1}{A(D)} \iint_D e^{y^3} dA = \frac{1}{8/3} \left(\frac{1}{3} \right) \left(e^8 - 1 \right) =$$

$$= \left(\frac{3}{8} \right) \left(\frac{1}{3} \right) \left(e^8 - 1 \right) = \frac{1}{8} \left(e^8 - 1 \right)$$