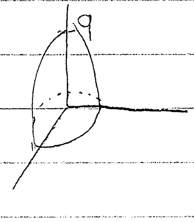
Quiz 6 solutions

1. 

The paraboloid intersects the xy-plane in the circle $x^2+y^2=9$ (set $z=0$ in the equation). In polar coordinates, the disk (and its interior) can be represented by $\{(r,\theta) \mid 0 \le r \le 3,\ 0 \le \theta \le 2\pi\}$. Since

$$\frac{\partial z}{\partial x} = -2x, \quad \frac{\partial z}{\partial y} = -2y,$$

we need to compute

$$\iint_R \sqrt{1+4x^2+4y^2}\, dA$$

where R is the disk given above. The integral is thus

$$\int_0^{2\pi}\int_0^3 r\sqrt{1+4r^2}\, dr\, d\theta.$$

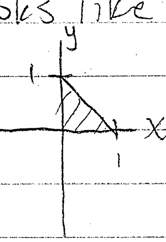
Let $u=1+4r^2$. Then $du/8 = r\,dr$, so that we have

$$\frac{1}{8}\int_0^{2\pi}\int_1^{37} u^{1/2}\, du\, d\theta = \frac{1}{8}\int_0^{2\pi} \frac{2}{3}u^{3/2}\Big|_{u=1}^{u=37} d\theta$$

$$= \frac{37\sqrt{37}-1}{12}\int_0^{2\pi} d\theta$$

$$= \frac{37\sqrt{37}-1}{6}.$$

2. The triangular region looks like



so E can be written as

$$E = \{(x,y,z) \mid 0 \le x \le 1,\ 0 \le y \le 1-x,\ 0 \le z \le x\},$$

which gives

$$\iiint_E x^2y\, dV = \int_0^1\int_0^{1-x}\int_0^x x^2y\, dz\, dy\, dx.$$