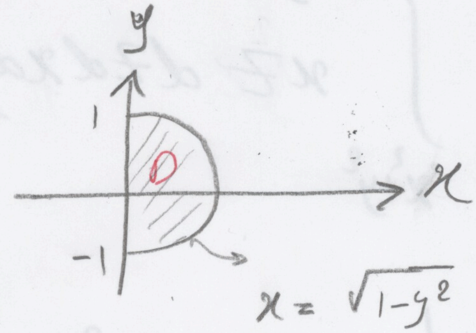
Quiz #7

1) The range of variables are

$$\left.\begin{array}{l} -1 \leq y \leq 1 \\ 0 \leq x \leq \sqrt{1-y^2} \end{array}\right\} \Rightarrow$$



$$x^2+y^2 \leq z \leq \sqrt{x^2+y^2}$$

Hence from the figure above

$$0 \leq r \leq 1 \quad , \quad -\pi/2 \leq \theta \leq \pi/2$$

Since $\quad x^2+y^2 = r^2 \implies r^2 \leq z \leq r$

Therefore

$$E = \left\{ (x,y,z) \;\middle|\; \begin{array}{c} -1 \leq y \leq 1 \\ 0 \leq x \leq \sqrt{1-y^2} \\ x^2+y^2 \leq z \leq \sqrt{x^2+y^2} \end{array} \right\} = \left\{ (r,\theta,z) \;\middle|\; \begin{array}{c} -\pi/2 \leq \theta \leq \pi/2 \\ 0 \leq r \leq 1 \\ r^2 \leq z \leq r \end{array} \right\}$$

Hence the integral in cylindrical coordinates becomes

$$\int_{-1}^{1}\int_{0}^{\sqrt{1-y^2}}\int_{x^2+y^2}^{\sqrt{x^2+y^2}} xz\, dz\, dx\, dy = \int_{-\pi/2}^{\pi/2}\int_{0}^{1}\int_{r^2}^{r} (r\cos\theta)\, z\, r\, dz\, dr\, d\theta$$

$$= \int_{-\pi/2}^{\pi/2}\int_{0}^{1} \left[r^2 \cos\theta \, \frac{z^2}{2} \right]_{z=r^2}^{z=r} dr\, d\theta$$

$$= \frac{1}{2}\int_{-\pi/2}^{\pi/2}\int_{0}^{1} (r^4 - r^6) \cos\theta \, dr\, d\theta =$$

$$= \frac{1}{2}\int_{-\pi/2}^{\pi/2} \cos\theta \, d\theta \int_{0}^{1} (r^4 - r^6)\, dr =$$

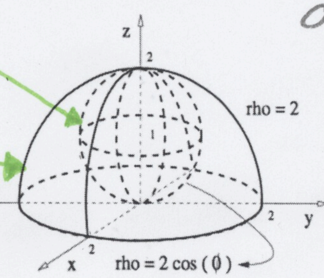
$$= \frac{1}{2}\left[\sin\theta\right]_{-\pi/2}^{\pi/2} \left[\frac{r^5}{5} - \frac{r^7}{7} \right]_{0}^{1} =$$

$$= \frac{1}{2}\left[1+1\right]\left[\frac{1}{5}-\frac{1}{7}\right] = \frac{1}{2}(2)\left(\frac{2}{35}\right) = \frac{2}{35}$$

Quiz #7

2) Since $\cos\phi = z/\rho \Rightarrow \rho = 2\cos\phi = 2z/\rho \Rightarrow \rho^2 = 2z \Rightarrow$
$x^2+y^2+z^2 = 2z \Rightarrow \underline{x^2+y^2+(z-1)^2 = 1}$ An sphere of radius 1 centered at $(0,0,1)$

$\underline{\rho = 2}$ is a sphere of radius 2



and $0 \le \phi \le \pi/2$, since

we only consider the upper half

of the sphere. Hence the domain E is

$$\left\{ (\rho, \theta, \phi) \,\middle|\, 0 \le \phi \le \pi/2,\ 0 \le \theta \le 2\pi,\ 2\cos\phi \le \rho \le 2 \right\}$$

Therefore the volume V of the region is.

$$V = \int_0^{2\pi} \int_0^{\pi/2} \int_{2\cos\phi}^{2} \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$$