Quiz # 8

1) $f(x(t), y(t)) = \frac{1}{16}(1+3t-1)(4t)^2 = 3t^3$

$ds = |\vec{r}'(t)|\, dt = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt =$

$= \sqrt{(3)^2 + (4)^2}\, dt = 5\, dt \Rightarrow \boxed{ds = 5\,dt}$

$\int_C f(x,y)\, ds = \int_0^1 3t^3 (5)\, dt = 15\int_0^1 t^3\, dt =$

$15\left[\frac{t^4}{4}\right]_0^1 = \frac{15}{14}$

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2) Let $P(x,y) = \sin y$ and $Q(x,y) = x\cos y$

then $\dfrac{\partial P}{\partial y} = \cos y = \dfrac{\partial Q}{\partial x}$

Also the domain of $\vec{F}$ is the entie plane $\mathbb{R}^2$ which is open and simply-connected, hence $\vec{F}$ is a conservative vector field

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If $\vec{F} = \nabla f \Rightarrow f_x = \sin y \Rightarrow f(x,y) = x\sin y + g(y)$

$\Rightarrow f_y = x\cos y + g'(y) = Q = x\cos y \Rightarrow$

$\Rightarrow g'(y) = 0 \Rightarrow \boxed{g(y) = K \text{ constant}}$

$\Rightarrow \boxed{f(x,y) = x\sin y + K}$

Hence $\displaystyle\int_C \vec{F}\cdot d\vec{r} = \int_C \nabla f\cdot d\vec{r} = f(\vec{r}(t_1)) - f(\vec{r}(t_0))$

Quiz #8

$$t_0 = \pi/6 \Rightarrow \vec{r}(\pi/6) = \sin\pi/6\,\vec{i} + \pi/6\,\vec{j} = 1/2\,\vec{i} + \pi/6\,\vec{j}$$

$$\Rightarrow \boxed{x_0 = 1/2 \;,\; y_0 = \pi/6} \Rightarrow$$

$$\Rightarrow f(\vec{r}(t_0)) \equiv f(x_0, y_0) = 1/2 \sin\pi/6 + K = 1/4 + K$$

$$t_1 = \pi/2 \Rightarrow \vec{r}(\pi/2) = \sin\pi/2\,\vec{i} + \pi/2\,\vec{j} = 1\,\vec{i} + \pi/2\,\vec{j}$$

$$\Rightarrow \boxed{x_1 = 1 \;,\; y_1 = \pi/2} \Rightarrow$$

$$f(\vec{r}(t_1)) \equiv f(x_1, y_1) = \sin\pi/2 + K = 1 + K$$

$$\Rightarrow \int_C \nabla f \cdot d\vec{r} = f(x_1, y_1) - f(x_0, y_0) = (1+K) - (1/4 + K)$$

$$= 3/4$$