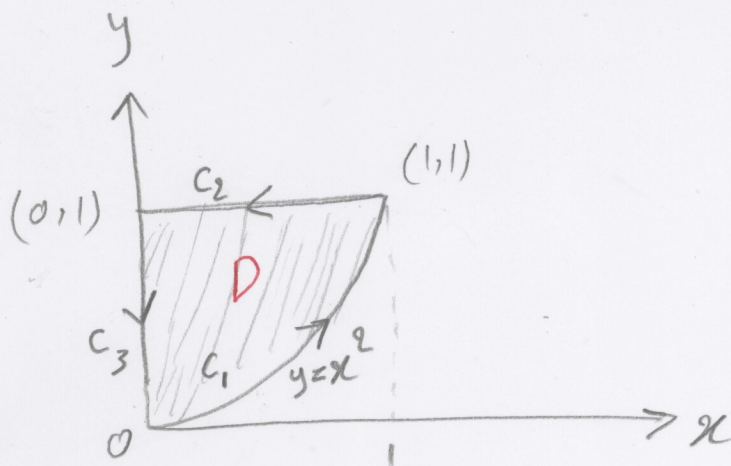


Quiz #9

1)



$$D = \{(x, y) \mid 0 \leq x \leq 1, x^2 \leq y \leq 1\}$$

$$\int x^2(y+1) dx + x\sqrt{y} dy = \iint_D \frac{\partial(x\sqrt{y})}{\partial x} - \frac{\partial(x^2(y+1))}{\partial y} dA$$

C By using Fubini's theorem

$$= \iint_D (\sqrt{y} - x^2) dA = \int_0^1 \int_{x^2}^1 (\sqrt{y} - x^2) dy dx =$$

$$= \int_0^1 \left[\frac{2}{3} y^{3/2} - x^2 y \right]_{y=x^2}^{y=1} dx = \int_0^1 \left(\frac{2}{3} - x^2 \right) - \left(\frac{2}{3} x^3 - x^4 \right) dx =$$

$$= \int_0^1 \left(\frac{2}{3} - x^2 - \frac{2}{3} x^3 + x^4 \right) dx = \left[\frac{2}{3} x - \frac{x^3}{3} - \frac{x^4}{6} + \frac{x^5}{5} \right]_0^1$$

$$= \frac{2}{3} - \frac{1}{3} - \frac{1}{6} + \frac{1}{5} = \frac{11}{30}$$

2)

$$\begin{aligned}
 \text{curl } \vec{F} &= \vec{\nabla} \times \vec{F} = \left(\frac{\partial}{\partial x} \vec{i} + \frac{\partial}{\partial y} \vec{j} \right) \times \left(e^{xy} \vec{i} + x \sin y \vec{j} \right) = \\
 &= \frac{\partial (x \sin y)}{\partial x} \vec{k} + \frac{\partial e^{xy}}{\partial y} (-\vec{k}) = \\
 &= \sin y \vec{k} - x e^{xy} \vec{k} = (\sin y - x e^{xy}) \vec{k}
 \end{aligned}$$

$$\begin{aligned}
 \text{div } \vec{F} &= \vec{\nabla} \cdot \vec{F} = \left(\frac{\partial}{\partial x} \vec{i} + \frac{\partial}{\partial y} \vec{j} \right) \cdot \left(e^{xy} \vec{i} + x \sin y \vec{j} \right) = \\
 &= \frac{\partial e^{xy}}{\partial x} + \frac{\partial (x \sin y)}{\partial y} = y e^{xy} + x \cos y
 \end{aligned}$$
