QUIZ 1

Problem. Show that $|\sin(\sum_{i=1}^{n} x_i)| \leq \sum_{i=1}^{n} |\sin(x_i)|$ for all n with x_i arbitrary real numbers. [Hint: Use the angle addition formula $\sin(x+y) = \sin(x)\cos(y)+\sin(y)\cos(x)$.]

Solution. We will procede by induction on n.

Case $n = 1$. It is trivially true that $|\sin(x_1)| = |\sin(x_1)|$ for any real number x_1.

Case $n + 1$. Assume by the inductive hypothesis that *Case* n holds. We have that:

$$\begin{aligned}
|\sin(\sum_{i=1}^{n+1} x_i)| &= |\sin(\sum_{i=1}^{n} x_i + x_{n+1})| \\
&\text{[by angle addition]} \\
&= |\sin(\sum_{i=1}^{n} x_i)\cos(x_{n+1}) + \sin(x_{n+1})\cos(\sum_{i=1}^{n} x_i)| \\
&\text{[by triangle inequality]} \\
&\leq |\sin(\sum_{i=1}^{n} x_i)\cos(x_{n+1})| + |\sin(x_{n+1})\cos(\sum_{i=1}^{n} x_i)| \\
&\leq |\sin(\sum_{i=1}^{n} x_i)| + |\sin(x_{n+1})|
\end{aligned} \tag{1}$$

since $-1 \leq \cos(x) \leq 1$ for any real number x.

By the inductive hypothesis the last line is at most

$$\sum_{i=1}^{n} |\sin(x_i)| + |\sin(x_{n+1})| = \sum_{i=1}^{n+1} |\sin(x_i)|$$

and we are done.