Quiz 2

Problem. Let (x_n) be a sequence converging to x such that $x_n \geq 0$ for all n. Show that $x \geq 0$.

Solution. We will procede by contradiction.

Suppose that $x < 0$. Pick $\epsilon > 0$ small enough that $(x-\epsilon, x+\epsilon) \subset (-\infty, 0)$. (Choosing $\epsilon = |x|/2$ will work, for instance.) By the definition of convergence there exists N such that for all $n \geq N$ we have that $|x_n - x| < \epsilon$. This implies that $x_N \in (x - \epsilon, x + \epsilon)$ so $x_N < 0$, a contradiction.