**MATH 341, QUIZ #4**
**FRIDAY, FEBRUARY 22**

**Question 0.0.1.** Use the $\epsilon$-$\delta$ definition of the limit to show $\lim_{x\to 2} x^2 + 1 = 5$

*Proof.* We have that $f(x) = x^2 + 1$, $L = 5$, and $c = 2$. So

$$|f(x) - L| = |(x^2 + 1) - 5| = |x^2 - 4| = |(x + 2)(x - 2)| = |x + 2| \cdot |x - 2|.$$

If $1 < x < 3$ (in other words, $|x - 2| < 1$), then $3 < |x + 2| < 5$, thus

$$|(x^2 + 1) - 5| \leqslant 5|x - 2|.$$

Given $\epsilon > 0$, choosing $\delta \leqslant \epsilon/5$ (and also $\delta < 1$), we have that whenever $0 < |x - 2| < \delta$,

$$|(x^2 + 1) - 5| < 5|x - 2| = 5\delta \leqslant \epsilon$$

thus verifying that $\lim_{x\to 2} x^2 + 1 = 5$. □