MA 351 Linear Algebra Homework 1

Due Friday, September 1

August 25, 2017

1 State whether each of the following statements is correct or incorrect. Explain why.

(a) A system of linear equations cannot have exactly 2 solutions.
(b) A system of n equations in n variables can have at most one solution.
(c) The rank of a system is less than or equal to the number of equations in the system.

2 For each linear system express the solution set in parametric form.

$$\text{(a)} \begin{cases} x+3y+z=1 \\ 2x+4y+7z=2 \\ 3x+10y+5z=7 \end{cases} ;\ \text{(b)} \begin{cases} x+3y+z=1 \\ 2x+4y+7z=2 \\ 4x+10y+9z=7 \end{cases} ;\ \text{(c)} \begin{cases} 2x-y+4z+w=3 \\ x-2y+3z=1 \\ 3y+z-2w=4 \end{cases}$$

3 A system is said to be homogeneous if the augmented matrix is of the form $[A|\vec{b}]$ where $\vec{b}=(0,...,0)^t$. Is a homogeneous system always consistent? Explain.

4 Give examples of systems of four linear equations in four variables whose solution sets are:

(a) a point; (b) a line; (c) a plane

5 Show that if $\vec{x}$ and $\vec{y}$ are solutions to the same linear system, then so is $\vec{x}/2+\vec{y}/2$. What about $\vec{x}+\vec{y}$? Explain.

6 Bring each of these matrices into reduced echelon form.

$$\text{(a)} \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 2 & 2 & 1 & 1 & 1 \\ 1 & 0 & 2 & 3 & 2 \\ 4 & 3 & 2 & 1 & 0 \end{bmatrix};\ \text{(b)} \begin{bmatrix} 3 & 1 & -1 & 4 \\ 1 & 0 & 2 & 1 \\ 1 & 1 & -1 & 2 \end{bmatrix};\ \text{(c)} \begin{bmatrix} a & b \\ c & d \end{bmatrix}, ad-bc=0$$

7 Show that a consistent system of n equations with n unknowns has an infinite number of solutions exactly when the rank of the system is less than n.

8

Let $A=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $\vec{x}$ any 2-vector. Show that $[A|\vec{x}]$ has exactly one solution when $ad-bc\neq 0$.