MATH 351, FALL 2017, HOMEWORK #10

DUE FRIDAY, DECEMBER 8

Problem 1 (8.3, p. 452)**.** Show for all $\vec{x}$ in $\mathbb{R}^n$ that the following hold:

a) $\|\vec{x}\|_\infty \leq \|\vec{x}\|$
b) $\|\vec{x}\| \leq \sqrt{n}\|\vec{x}\|_\infty$
c) $\|\vec{x}\|_1 \leq \sqrt{n}\|\vec{x}\|$

Problem 2 (8.10, p. 452)**.** Show that for an $n \times n$ orthogonal matrix A that $\operatorname{cond}(A) \leq n$.

Problem 3. Let A be a symmetrix real $n \times n$ matrix wth eigenvalues $|\lambda_1| > |\lambda_2| \geq |\lambda_3| \geq \cdots \geq |\lambda_k| \geq 0$. Let W be the orthogonal complement of the span of the λ_1 eigenvectors. Show that $|\lambda_2|$ is the maximum of the ratios

$$\frac{\|A\vec{x}\|}{\|\vec{x}\|}$$

where $\vec{x}$ is a non zero vector belonging to W.

Problem 4. Let A be an $n \times n$ real matrix with non-negative entries such that the entries in each row sum up to 1. Show that 1 is an eigenvalue of A. What about if the entries in the columns sum to 1 instead?

Problem 5. Show that if A and B are both invertible $n \times n$ real matrices, then AB and BA have the same eigenvalues.

Problem 6 (5.31, p. 291)**.** Let A be an $n \times n$ real matrix with distinct eigenvalues $\lambda_1, \ldots, \lambda_n$. Show that $\det(A) = \lambda_1 \cdots \lambda_n$.

Problem 7 (cf. 5.36, p. 291)**.** Let A be a real $n \times n$ matrix, and let $p(\lambda) = \det(A - \lambda I)$ be its characteristic polynomial. If A is diagonalizable show that $p(A) = 0$.

Problem 8 (4.16, p. 259)**.** Show that

$$\det \begin{pmatrix} 1 & 1 & 1 \\ x & y & z \\ x^2 & y^2 & z^2 \end{pmatrix} = (x-y)(y-z)(z-x)$$

Problem 9 (4.39, p. 269)**.** Use Theorem 4.15 on p. 267 to compute the inverse of the matrix

$$\begin{bmatrix} 5 & 2 & 1 \\ 2 & 2 & 2 \\ 2 & 1 & 1 \end{bmatrix}.$$