# MA 351 Linear Algebra Homework 2

**<u>Due on Friday, September 8</u>**

September 1, 2017

1 Determine the truth of each statement and explain.

(a) A subset of a linearly independent set is linearly independent.
(b) A subset of a linearly dependent set is linearly dependent.
(c) A set that contains a linearly independent set is linearly independent.
(d) For any set of linearly dependent $m \times n$ matrices, the set consisting of its transposes is a linearly dependent set of $n \times m$ matrices.

2 For a vector space $V$ show that.

(a) Any vector $\vec{x}$ has a unique additive inverse $-\vec{x}$.
(b) $0 \cdot \vec{x} = \vec{0}$.
(c) $(-1) \cdot \vec{x} = -\vec{x}$.

3 Let $S$ be the collection of all $3 \times 3$ real matrices $A$ such that $A^t = -A$. Show that S is a vector space. Find a spanning set of three vectors. Let $V_n$ be the set of all vectors $[x_1, \ldots, x_n]^t$ in $\mathbb{R}^n$ such that $\sum_{i=1}^n x_i = 0$. Is $V_n$ a vector space? If so, how many vectors are in the smallest spanning set?

4 Show that $\{e^x, e^{2x}, e^{3x}\}$ is a linearly independent set. What about $\{\sin(x), \sin(2x), \sin(3x)\}$?

5 Find four linearly independent vectors in $\mathbb{R}^4$ all of whose entries are non-zero.

6 Let $\vec{x}_0 + t_1\vec{x}_1 + t_2\vec{x}_2 + t_3\vec{x}_3$ be a parametric form of the solution set to a system of linear equations. Is $\{\vec{x}_1, \vec{x}_2, \vec{x}_3\}$ a linearly independent set? Explain. How about $\{\vec{x}_0, \vec{x}_1, \vec{x}_2, \vec{x}_3\}$?

7 Let $\vec{x} = [1, -2, 4]^t$ and $\vec{y} = [-1, 2, 3]^t$ .

Are there any elements in their span with all entries positive? Explain.

8 Let $\vec{y}_1, \vec{y}_2, \ldots, \vec{y}_k$ all belong to the span of $\{\vec{x}_1, \vec{x}_2, \ldots, \vec{x}_n\}$.

Show that any linear combination of these vectors also belongs to the span of $\{\vec{x}_1, \vec{x}_2, \ldots, \vec{x}_n\}$.