MATH 351, FALL 2017, HOMEWORK #3

<u>**DUE FRIDAY, SEPTEMBER 15**</u>

Problem 1. True or false? Explain.

a) The row vectors of a matrix in echelon form are linearly independent as long as each row vector is non-zero.
b) The column vectors of a matrix in echelon form are linearly independent as long as each column vector is non-zero.

Problem 2. Find a map $T : \mathbb{R}^2 \to \mathbb{R}^2$ so that $T(\vec{x} + \vec{y}) = T(\vec{x}) + T(\vec{y})$ for all $\vec{x}, \vec{y} \in \mathbb{R}^2$ but which is not linear.

Problem 3. Find a map $T : \mathbb{R}^2 \to \mathbb{R}^2$ so that $T(c \cdot \vec{x}) = c \cdot T(\vec{x})$ for all $\vec{x} \in \mathbb{R}^2$ and all $c \in \mathbb{R}$ but which is not linear.

Problem 4. Find a map $T : \mathbb{C}^2 \to \mathbb{C}^2$ so that $T(c \cdot \vec{x}) = c \cdot T(\vec{x})$ for all $\vec{x} \in \mathbb{C}^2$ and all $c \in \mathbb{R}$ but which is not linear. (We are regarding $\mathbb{C}^2$ as a vector space over $\mathbb{C}$.)

Problem 5. Let W_1 and W_2 be subspaces of the same vector space V. We define the set $W_1 + W_2$ as follows:

$$W_1 + W_2 = \{\vec{x} + \vec{y} : \vec{x} \in W_1 \text{ and } \vec{y} \in W_2\}.$$

Show that $W_1 + W_2$ is a subspace of V.

Problem 6. Let W_1 and W_2 be subspaces of the same vector space V. We define the set $W_1 \cap W_2$ as follows:

$$W_1 \cap W_2 = \{\vec{x} : \vec{x} \in W_1 \text{ and } \vec{x} \in W_2\}.$$

Show that $W_1 \cap W_2$ is a subspace of V.

What about $W_1 \cup W_2 = \{\vec{x} : \vec{x} \in W_1 \text{ or } \vec{x} \in W_2\}$?

Problem 7. Find the null space of each of the following matrices.

a)

$$A = \begin{bmatrix} 1 & 1 & -1 \\ 1 & -1 & 1 \\ -1 & 1 & 1 \end{bmatrix}$$

b)

$$B = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 3 & 4 & -2 & 0 \\ 0 & 0 & 0 & -3 & 1 \end{bmatrix}$$

c)

$$C = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 2 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

Problem 8. Find a vector $\vec{b}$ such that $A\vec{x} = \vec{b}$ is inconsistent if one exists.

a)

$$A = \begin{bmatrix} 1 & 1 & -1 \\ 1 & -1 & 1 \\ -1 & 1 & 1 \end{bmatrix}$$

b)

$$A = \begin{bmatrix} 1 & 3 \\ 2 & 2 \\ -1 & 1 \\ -2 & 0 \end{bmatrix}$$

c)

$$A = \begin{bmatrix} 1 & 1 & -5 & 8 & 7 \\ -1 & 2 & 0 & -7 & 6 \\ 0 & 3 & 10 & 0 & 0 \\ 0 & -6 & -5 & -1 & -13 \end{bmatrix}$$