MATH 351, FALL 2017, HOMEWORK #5

DUE FRIDAY, OCTOBER 6, 2017

Problem 1 (cf. 3.64, p. 190)**.** Invert the following matrices, if possible.

a)
$$\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

b)
$$\begin{bmatrix} 2 & -1 & 2 & 0 \\ 4 & -1 & 4 & -2 \\ 8 & -3 & 10 & 0 \\ 6 & -3 & 8 & 9 \end{bmatrix}$$

c)
$$\begin{bmatrix} 5 & -1 & 2 \\ 1 & 0 & 1 \\ 6 & 0 & 3 \end{bmatrix}$$

Problem 2 (cf. 3.74, p. 192)**.** . A matrix $N \in M_{nn}(\mathbb{R})$ is said to be **nilpotent** of degree k if $N^k = 0$.

a) If N is nilpotent of degree k, show that
$$(I - N)^{-1} = I + N + N^2 + \cdots + N^{k-1}.$$

b) Use this to invert the matrix
$$\begin{bmatrix} 1 & -2 & 3 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix}$$

Problem 3 (3.77, p. 192)**.** If A is a square matrix such that $A^2+3A+I = 0$, show that $-A - 3I$ is an inverse for A.

Problem 4 (3.80 and 3.84, p. 193)**.** If $A, B \in M_{nn}(\mathbb{R})$ are invertible matrices.

a) Show that AB is again invertible.
b) Show that $(A^{-1})^t = (A^t)^{-1}$.

Problem 5 (cf. 2.3, p. 109)**.** Find a basis for $\mathrm{Col}(A)$.

a)

$$\begin{bmatrix} 1 & 2 & -8 & 4 \\ 2 & 1 & -1 & -1 \\ -3 & 3 & -21 & 15 \\ 3 & 4 & -14 & 6 \end{bmatrix}$$

b)

$$\begin{bmatrix} 2 & 3 & -1 & 19 & 2 \\ -1 & 0 & 2 & -8 & 5 \\ 2 & -1 & -5 & 15 & -14 \end{bmatrix}$$

c)

$$\begin{bmatrix} 3 & 1 & 2 & -7 \\ 2 & 4 & 0 & 8 \\ -1 & -2 & -1 & -2 \\ 2 & 2 & 3 & 8 \end{bmatrix}$$

Problem 6 (cf. 2.10, p. 111)**.** Let $A \in M_{kn}(\mathbb{R})$. Is there relationship between the rank of the homogeneous system of linear equations given by $A \cdot \vec{x} = \vec{0}$ and the number of vectors in a basis for $\mathrm{Col}(A)$?. Explain.

Problem 7 (2.20, p. 112)**.** Show that the columns of a matrix are linearly independent exactly when the null space of A is $\{\vec{0}\}$.

Problem 8 (cf. 2.41, p. 125)**.** Consider $\mathbb{R}^n$

a) Find a basis for $\mathbb{R}^4$ containing the vectors $(1, 2, 3, 4)^t$ and $(1, 0, 1, 0)^t$.
b) Given k linearly independent vectors in $\mathbb{R}^n$, is it always possible to find a basis for $\mathbb{R}^n$ containing those vectors? Explain.