MATH 351, FALL 2017, HOMEWORK #7

DUE FRIDAY, OCTOBER 27, 2017

Problem 1. Show that a non-zero nilpotent matrix is not diagonalizable.

Problem 2 (5.36, p. 291)**.** If A is diagonalizable with eigenvalues $\lambda_1, \dots, \lambda_k$ and $q(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$ is a polynomial such that $q(\lambda_i) = 0$ for all $i = 1, \dots, k$, show that

$$q(A) = a_n A^n + a_{n-1}A^{n-1} + \cdots + a_1 A + a_0 I = 0.$$

Problem 3. cf. 5.37] Find all values of a, b, c so that

$$\begin{bmatrix} 2 & a & b \\ 0 & -5 & c \\ 0 & 0 & 2 \end{bmatrix}$$

is diagonalizable.

Problem 4 (cf. 6.4 and 6.5, p. 317)**.** Geometrically describe each set.

a) For $\vec{x} = [1, 1]^t$, the set $\{\vec{y} \in \mathbb{R} : \vec{x} \cdot \vec{y} \leq \|\vec{y}\|\}$.
b) For $\vec{x} = [1, 1, 1]$, the set $\{\vec{y} \in \mathbb{R}^3 : \vec{x} \cdot \vec{y} \geq \frac{3}{2}\|\vec{y}\|\}$.

Problem 5 (cf. 6.11, p. 318)**.** For an orthogonal set of vectors $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ in $\mathbb{R}^4$, show that there is a vector $\vec{v}_4$ so that $\{\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4\}$ forms an orthogonal basis for $\mathbb{R}^4$.

Problem 6 (cf. 6.14, p. 318)**.** Show the Cauchy-Schwart inequality

$$-\|\vec{x}\|\|\vec{y}\| \leq \vec{x} \cdot \vec{y} \leq \|\vec{x}\|\|\vec{y}\|$$

for all $\vec{x}, \vec{y} \in \mathbb{R}^n$.

Hint: Show that for any real number R

$$0 \leq \|\vec{x} + R\vec{y}\|^2 = \|\vec{x}\|^2 + 2\|\vec{x}\|\|\vec{y}\|R + \|\vec{y}\|^2 R^2,$$

so this quadratic polynomial in R has at most one root.

Problem 7 (cf. 6.17, p. 318)**.** Show that any orthogonal set of vectors $\{\vec{v}_1, \dots, \vec{v}_k\}$ in $\mathbb{R}^n$ is linearly independent.