MATH 351, FALL 2017, HOMEWORK #8

DUE FRIDAY, NOVEMBER 3, 2017

Problem 1. Write down an orthogonal basis for $\mathbb{R}^4$ using only the numbers 1 and -1 as entries for the vectors.

Problem 2 (6.28, p.330)**.** For any subset S of $\mathbb{R}^n$ show that

$$S^{\perp} = \{\vec{x} \in \mathbb{R}^n : \vec{x} \cdot \vec{s} = 0 \text{ for all } \vec{s} \in S\}$$

is a subspace.

Problem 3. For any subspace W of $\mathbb{R}^n$, show that $(W^{\perp})^{\perp} = W$.

Problem 4 (cf. 6.22, p. 329)**.** Use the Gram-Schmidt process to find an orthogonal basis for the subspace of $\mathbb{R}^n$ spanned by the following vectors.

a) $\{[1,2,1,2]^t, [1,2,3,0]^t, [1,1,-1,-1]^t\}$
b) $\{[1,1,1,1,1]^t, [1,1,0,1,-1]^t, [1,-1,0,0,1]^t\}$

Problem 5 (6.20, p. 330)**.** Find an orthogonal basis for $S^{\perp}$ for each of these sets:

a) $S = \{[1,3,1,-1]^t, [2,6,0,1]^t, [4,12,2,-1]^t\}$
b) $S = \{[1,3,1,-1]^t, [2,6,0,1]^t\}$
c) $S = \{[1,1,1,1,1]^t, [1,1,0,1,-1]^t, [1,-1,0,0,1]^t\}$

Problem 6 (6.27, p. 330)**.** Let A be a $k \times n$ real matrix, and let W be the null space of A. Show that $A\vec{x} = A(P_{W^{\perp}}(\vec{x}))$ for any $\vec{x} \in \mathbb{R}^n$, where $P_{W^{\perp}}(\vec{x})$ is the orthogonal projection onto $W^{\perp}$.

Problem 7 (cf. 6.39, p. 331)**.** Let $\vec{x}, \vec{y} \in \mathbb{R}^n$ be non zero vectors. Show that the orthogonal projection of $\vec{x}$ onto $\mathbb{R}\vec{y}$ is the vector of closest distance to $\vec{x}$ belonging to $\mathbb{R}\vec{y}$. Show this by:

a) minimizing the function $f(t) = \|\vec{x} - t\vec{y}\|^2$
b) using that for any subspace W of $\mathbb{R}^n$, $\|\vec{x} + \vec{y}\|^2 = \|\vec{x}\|^2 + \|\vec{y}\|^2$ for $\vec{x} \in W$ and $\vec{y} \in W^{\perp}$.