MATH 351, FALL 2017, HOMEWORK #9

DUE FRIDAY, NOVEMBER 10, 2017

Problem 1 (cf. 6.120, p. 394)**.** Let A be an $n \times n$ real matrix such that $A = A^t$. We will say that A is positive semidefinite if $(A\vec{x}) \cdot \vec{x} \geq 0$ for all $\vec{x} \in \mathbb{R}^n$.

Show that if $A = B^t B$ for some real $k \times n$ matrix B, then A is positive semidefinite.

Problem 2. Show the following properties are equivalent to $A = A^t$ being positive semidefinite:

a) All eigenvalues of A are non-negative.
b) There is an $n \times n$ real matrix C such that $C = C^t$ and $A = C^2$.

Problem 3 (cf. 6.118, p. 394)**.** Find a 2×2 real matrix, equal to its transpose, with positive entries but which is not positive semidefinite.

Problem 4. Let P be an $n \times n$ real matrix such that $P^t = P$ and $P^2 = P$. Show that P is the matrix of the orthogonal projection onto the subspace of all vectors $\vec{x} \in \mathbb{R}^n$ such that $P\vec{x} = \vec{x}$.