MATH 490, WORKSHEET #1
WEDNESDAY, FEBRUARY 6

Problem 1. Let a, b, c be side lengths of a triangle. Show that $3(ab + bc + ca) \leqslant (a + b + c)^2 \leqslant 4(ab + bc + ca)$.

Problem 2. Show that for $x_1 < \cdots < x_n$ and $\epsilon_i = \pm 1$ the set of all possible sums $\sum_{i=1}^{n} \epsilon_i x_i$ contains at least $\binom{n+1}{2}$ distinct values.

Problem 3. Every positive integer is a sum of distinct Fibonacci numbers.

Problem 4. A fair coin is flipped n times. What is the probability of two heads appearing in succession?

Problem 5. A 4×7 chess board must contain a rectangle whose corner squares are all the same color.

Problem 6. Show that there are integers a, b, c, not all zero so that

$$|a + \sqrt{2}b + \sqrt{3}c| < 10^{-11}.$$

Problem 7. 15 dinner guests are to be seated around a circular table. Each seat has a name card, but everyone sits in such a way that no one is seated in their assigned seat. Show the table may be rotated so that at least 2 guests are seated at their assigned seat.

Problem 8, a Classic. Among 6 people there are either 3 mutual friends or 3 mutual strangers.

Problem 9. How many invertible $\{0, 1\}$ valued $n \times n$ matrices are there?