MATH 490, WORKSHEET #2
WEDNESDAY, FEBRUARY 13

Problem 1. Choose 55 numbers from the set $\{1, 2, \ldots, 100\}$. Show that this set must contain numbers differing by 10, 12, and 13, but not necessarily differing by 11.

Problem 2. Show that no prime p of the form $p = 4k + 3$ is a sum of two squares.

Problem 3. Show that $11, 111, 1111, \ldots$ contains no square numbers.

Problem 4. Show that if n divides a Fibonacci number, it divides infinitely many Fibonacci numbers.

Problem 5. If ab, ac, and bc are perfect cubes, so are a, b, and c.

Problem 6. Suppose the prime factorizations of $r + 1$ integers only involve a total of r primes. Show there is suset of these numbers whose product is a perfect square.

Problem 7. Show that $H_n = 1 + 1/2 + \cdots + 1/n$ is not an integer for any $n > 1$. (Hint: multiply both sides by $\operatorname{lcm}(1, \ldots, n)$ and work $\mod 2$.)

Problem 8, ICMC 2005. If 25 divides $x^3 + y^3 + z^3$, it divides at least one of $x^3 + y^3$, $y^3 + z^3$ or $x^3 + z^3$.