MATH 490, WORKSHEET #4
WEDNESDAY, FEBRUARY 27

Problem 1, ICMC 1990. A digraph in a word is an ordered pair of consecutive letters. So a word in n letters has $n-1$ digraphs. How many ways can INDIANA be rearranged so that no digraph is repeated? For instance IANDNAI is fine, but IANDNIA is not.

Problem 2, ICMC 1966 Show that the equation $x^2 - y^2 = a^3$ has integer solutions whenever a is a positive integer.

Problem 3, Putnam 2018. Find all pairs of integers (a,b) so that $\frac{1}{a} + \frac{1}{b} = \frac{3}{2018}$.

Problem 4, ICMC 1983. Show that $x^n + y^n = z^n$ has no solutions for $z < n$.

Problem 5, ICMC 2017. A 3×3 matrix of integers is called multiplicatively magic if the product of entries in any row or column or either of the two main diagonals is the same number, which we call the magic product. Show the magic product is a perfect cube and find the minimum magic product among all multiplicatively magic 3×3 matrices.

Problem 6, ICMC 1992. Find all right triangles with integer side lengths whose legs are consecutive integers. Hint: find a Pell equation.

Problem 7. Evaluate $\sum_{n=1}^{\infty} \frac{2n}{n(n+1)(n+2)}$.

Problem 8, Putnam 2016. Let $x_0 = 1$ and $x_{n+1} = \ln(\exp(x_n) - x_n)$ show that $\sum_{n=0}^{\infty} x_n$ converges and evaluate.

Problem 9, Putnam 2017. Let N be a positive integer $N = a + (a+1) + \cdots + (a+k-1)$ for $k = 2017$ but for no other integer $k > 1$. Considering all such N, what is the minimal value of a?