MATH 490, WORKSHEET #5
WEDNESDAY, MARCH 6

Problem 1, ICMC 2012. How many zeroes does the number 213! start with?

Problem 2, Putnam 1968 Show that $\int_0^1 \frac{x^4(1-x)^4}{1+x^2}\,dx = 22/7 - \pi$.

Problem 3, Putnam 1971. Find all functions which satisfy the identity $f(x) + f(1 - \frac{1}{x}) = 1 + x$ for all $x \neq 0, 1$.

Problem 4, ICMC 2016. Describe all functions $f : \mathbb{Z} \to \mathbb{Z}$ satisfying: (a) $f(2) = 2$; (b) $f(mn) = f(m)f(n)$; and (c) $f(m) > f(n)$ if $m > n$ for all $m, n \in \mathbb{Z}$.

Problem 5, ICMC 2012. If $p(x) = a_n x^n + \cdots + a_1 x + a_0$ is a polynomial with integer coefficients and a_0, a_1, a_n, and $a_2 + \cdots + a_n$ are all odd, then $p(x)$ has no rational root.

Problem 6, ICMC 2016. Find 8 points in 3-space such that all 56 triples of points form isosceles triangles.

Problem 7. ICMC 2015. f is a twice differentiable function $f(0) = f'(0) = 0$ and $f(1) = 1$, then there is $0 < a < 1$ so that $f'(a)f''(a) = 9/8$.

Problem 8, Putnam 1973. Let S consist of $2n + 1$ (possibly not distinct) integers for some n. S has the property that removing any member, the remaining can be divided into two sets of n with the same sum. Show that the numbers belonging to S are all equal.