MATH 490, WORKSHEET #7
WEDNESDAY, MARCH 27

Problem 1, ICMC 1999. Let P be any point inside an equilateral triangle T. Show that the sum of the three distances from P to the sides of T is constant.

Problem 2, IMOmath.org. Every point in the plane is painted in blue or red. Show that there are either two blue points at the distance exactly one, or four collinear red points such that the distances between any to consecutive is exactly 1.

Problem 3, ICMC 1996. Find the largest possible area of a pentagon with five sides of length 1 and a right interior angle.

Problem 4, Putnam 1988. The plane is divided into 3 disjoint sets. Can we always find two points in the same set a distance 1 apart? What about 9 disjoint sets?

Problem 5, ICMC 2004. Let P be the center of a square with side AC. Let B be a point in the exterior of the square such that ΔABC is a right triangle with hypotenuse AC. Prove: BP bisects $\angle ABC$.

Problem 6, Yaglom[2]. What is the greatest number of parts into which the plane can be divided by n lines? How about n circles?

Problem 7, VTRMC 2009. Two circles α, β touch externally at the point X. Let A, P be two distinct points on α different from X, and let AX and PX meet β again in the points B and Q respectively. Prove that AP is parallel to QB.