MATH 490, WORKSHEET #9
WEDNESDAY, APRIL 10

Problem 1, Putnam 1961. The set of pairs of positive real numbers (x, y) such that $x^y = y^x$ form a straight line $y = x$ and a curve. Find the point at which the curve intersects the line.

Problem 2, Putnam 2009. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be a function so that for every square in plane with vertices A, B, C, D, $f(A) + f(B) + f(C) + f(D) = 0$. Is is true that $f(P) = 0$ for all points in the plane?

Problem 3, Putnam 2003. Given n, how many ways can we write it as a sum of one or more positive integers $a_1 \leqslant a_2 \leqslant \ldots \leqslant a_k$ with $a_k \leqslant a_1 + 1$?.

Problem 4, Putnam 1967. Find the smallest positive integer n such that we can find a polynomial $nx^2 + ax + b$ with integer coefficients and two distinct roots in the interval $(0, 1)$.

Problem 5, Putnam 2008. Find the radius of a largest circle contained in the four-dimensional hypercube.

Problem 6, Putnam 1994. For which real numbers α does the graph of

$$y = x^4 + 9x^3 + \alpha x^2 + 9x + 4$$

contain four collinear points?

Problem 7, Putnam 2014. A sequence (a_n) of non-negative reals satisfies $a_{n+m} \leqslant a_n a_m$ for all positive integers m, n. Show that $\lim_{n\to\infty} a_n^{1/n}$ exists.

Problem 8, Putnam 1985. G is a finite group consisting of real $n \times n$ matrices under the operation of matrix multiplication. If the sum of the traces of the elements of G is zero, show that the sum of the elements of G is the zero matrix.