Syllabus Math 554, Fall 2011

1. Modules over an arbitrary commutative ring A. Kernel and image, direct sums and products, direct factors, definition of exact sequences, exactness properties of the $\mathrm{Hom}_A(M, \cdot)$ and $\mathrm{Hom}_A(\cdot, N)$ functors.

2. Definitions and main properties of projective and free modules. Dual modules and double duals. Canonical map c_E: $E \to E^{**}$. Properties in the special case of finitely generated projective modules. Basis free definition of trace of an endomorphism.

3. Definition and first properties of tensor products, tensor, symmetric and exterior algebras. Exactness property of $\cdot \otimes_A M$ The graded isomorphisms $\mathrm{Sym}(\bigoplus_{i\in I} E_i) \simeq \bigotimes_{i\in I} \mathrm{Sym}\, E_i$ and $\bigwedge(\bigoplus_{i\in I} E_i) \simeq \bigotimes_{i\in I} \bigwedge E_i$. Basis free definition of the determinant and its properties. Cauchy-Binet and Laplace formulas for the determinant. Co-transpose and Cramer's rule. The characteristic polynomial of an endomorphism and interpretation of its coefficients as traces of exterior powers.

4. Modules over principal ideal domains. Main theorem about f.g. modules over a PID. Application in the case of module over a fixed endomorphism of a finite dimensional vector space. Invariant factors of an endomorphism and how to calculate these. Minimal polynomial. Rational canonical form and (additive and multiplicative) Jordan decomposition theorem. Eigenvalues and eigenvectors. Simultaneous diagonalizability.

5. Inner product spaces over $\mathbb{C}$. Gram-Schmidt orthogonalization. Definition of the adjoint an endomorphism. Definition and properties of self-adjoint and unitary operators. The spectral theorem for self-adjoint operators. Definition of normal operators.

Textbooks: The main references are as follows:

1. nos. 1-3: *Algebra* Bourbaki, Chapters 2 and 3,
2. no. 4: *Algebra*, Bourbaki, Chapter 7,
3. no. 5: *Linear Algebra* by Hoffman and Kunz, Chapter 8.

There will be weekly homeworks, a mid-term and a final exam.