Analytic Number Theory: a first course
Instructor: Professor Trevor Wooley
Course Number: MA 59800AANT
Credits: Three
Time: 10:30 - 11:20 AM MWF

Description
This course serves as an introduction to analytic number theory. Its focus is the theory of the Riemann zeta-function and Dirichlet L-functions, the distribution of prime numbers, and Dirichlet's theorem on primes in arithmetic progressions. The development of the theory and application of Dirichlet series in number theory leads to surprisingly powerful results on the distribution of prime numbers, and today motivates a complex and beautiful body of research which aims to describe and explain arithmetic phenomena. This is a basic introduction to the theory of prime numbers and L-functions. Students interested in more advanced topics, or in preparing to undertake research in this area, will find this a useful first course ... and there are many beautiful results and theoretical developments along the way to keep the competent enthusiast interested.

Prerequisites in number theory will be confined to such topics as the Chinese remainder theorem and primitive roots (a basic first course will suffice). An analytic prerequisite is the calculation of contour integrals by summing residues. Further analytic material will be developed in the course ... Riemann-Stieltjes integration, Euler-Maclaurin summation, Jensen's inequality, the Borel-Caratheodory lemma, Hadamard products, and the Poisson summation formula.

Assessment: Six or seven (short) problem sets will be offered through the semester, and class participants can demonstrate engagement with the course by any written and/or in-class presentations featuring a reasonable subset of these problems – three levels of difficulty: short problems testing basic skill-sets, extended problems integrating the essential methods of the course, and more challenging problems for enthusiasts with detailed hints available on request.


Basic Topics. (i) Basic properties of Dirichlet series; (ii) Mean values of arithmetic functions, Renyi's theorem; (iii) Elementary prime number estimates; (iv) Simplest sieve estimates, the Brun-Titchmarsh theorem; (v) Dirichlet characters; (vi) Dirichlet L-functions; (vii) Primes in arithmetic progressions; (viii) The Mellin transform; (ix) Zeros of the zeta function; (x) The prime number theorem; (xi) Equivalent forms of the prime number theorem; (xii) Further comments on the prime number theorem; (xiii) Primitive characters, Gauss sums; (xiv) The Polya-Vinogradov inequality, primitive roots in short intervals; (xv) The Poisson summation formula; (xvi) Theta functions; (xvii) The functional equation of the zeta function and L-functions; (xviii) Zero-free regions for L-functions; (xix) Siegel's theorem, Landau's variant; (xx) The prime number theorem for arithmetic progressions and applications.

Advanced topics, depending on demand and available time, may include explicit formulae, applications of the Riemann Hypothesis, the large sieve and applications, estimates for prime number sums, the Bombieri-Vinogradov theorem.

Prerequisites: Elementary number theory and basic real and complex analysis.