Class Meeting Times: Tuesdays and Thursdays 10:30 - 11:45 in Math 215
Credit Hours: 3 hours
Course web page:

https://www.math.purdue.edu/~twooley/2020aha/2020aha.html

Prerequisites: Elementary number theory and basic analysis.

Instructor: Prof. Trevor Wooley, twooley@purdue.edu
Location: 422 Math, Tel. 765-496-6439
Office Hours: Tu 14:00-15:00, W 13:00-14:00, Th 14:30-15:30 in 422 Math

Course Description: This course serves as an introduction to the (Hardy-Littlewood) circle method, an important theme in analytic number theory, and complements the course MA59800 Section 76 by Tess Anderson on discrete analysis – taking them in parallel, an option that is encouraged but not required, would enhance the level of understanding of each of them. Background results from number theory and harmonic analysis will be reviewed as needed.

The (Hardy-Littlewood) circle method applies Fourier analysis to count rational or integral solutions of an equation or inequality in a manner respecting the inherent arithmetic. Developments in recent years have broadened its impact into additive combinatorics and discrete harmonic analysis beyond its more traditional role in quantitative arithmetic geometry. A highlight from the past five years is the full resolution of the Main Conjecture in Vinogradov’s mean value theorem.

We shall take as our central example Waring’s problem – the problem of understanding the number of representations of an integer as the sum of a fixed number of k-th powers of positive integers. Our aims are twofold: (i) to understand the scope and limitations of the circle method, and (ii) to gain some facility to apply the method, so from time to time there will be technical material that we’ll just cite rather than prove in any detail. This course is intended to be accessible to those without any background in analytic number theory.

Course Content:
(i) Discussion of Weyl’s inequality, Hua’s Lemma, and the simplest treatment of Waring’s problem. This provides an opportunity to discuss the key elements of the major arc analysis, that is, the singular integral and singular series, that together constitute the product of local densities. Density of integral zeros of diagonal equations.
(ii) Refinements to the major arc analysis, including use of Poisson summation. Sketch of Kloosterman method. Diminishing ranges. Diophantine equations arising as sums of binary forms.

(iii) Vinogradov’s methods, especially Vinogradov’s mean value theorem and ensuing analogue’s of Weyl’s inequality. Application to Waring’s problem and diagonal equations. The Main Conjecture and its consequences.


(v) Unrepresentation theory – the theory of exceptional sets of integers that fail to be represented in a specified form.

Learning outcomes: Students completing the course will: (i) acquire basic skills in an important theme of analytic number theory; (ii) gain experience applying analytic methods, and in particular harmonic analysis, to solve Diophantine problems; and (iii) be equipped to estimate exponential sums via the principal basic methods in the subject.

Course texts: The course will be based on the instructor’s lecture notes. Good texts for background reading and support are:


(b) H. Davenport, Analytic methods for Diophantine equations and Diophantine inequalities, Ann Arbor Publishers, Ann Arbor, 1962 or the LaTeXed version published by Cambridge University Press in 2005 [Friendlier for the basics, with material on general homogeneous cubics, but misses modern developments.]

(c) M. Nathanson, Additive number theory. The classical bases, GTM 164, Springer-Verlag, New York, 1996 [Pedestrian approach to the basics in which no corner is cut – good for getting started!]

Assessment: Course credit will be based solely on six (short) problem sets offered through the semester, posted on the course web-page:

https://www.math.purdue.edu/~twooley/2020aha/2020aha.html

Class participants can demonstrate engagement with the course by any written and/or in-class presentations featuring a reasonable subset of these problems. There are three levels of difficulty: short problems testing basic skill-sets, extended problems integrating the essential methods of the course, and more challenging problems for enthusiasts with detailed hints available on request. Working with other class members is permitted, but do write up the solutions individually by yourselves.

Boilerplate Notes for Boilermakers:

Attendance: In the event of an absence for medical or other reasons, students should review and follow the Student Regulations concerning Classes, informing the instructor as far in advance as possible.

https://www.purdue.edu/studentregulations/regulations_procedures/classes.html
**Academic Integrity:** Academic integrity is one of the highest values that Purdue University holds. Individuals are encouraged to alert university officials to potential breaches of this value by either emailing integrity@purdue.edu, or by calling 765-494-8778, or by contacting the Office of the Dean of Students (https://www.purdue.edu/odos/). While information may be submitted anonymously, the more information that is submitted provides the greatest opportunity for the university to investigate the concern. Purdue prohibits “dishonesty in connection with any University activity. Cheating, plagiarism, or knowingly furnishing false information to the University are examples of dishonesty” (Section B.2.a of the Student Regulations https://www.purdue.edu/studentregulations/student_conduct/regulations.html).

Furthermore, the University Senate has stipulated that “the commitment of acts of cheating, lying, and deceit in any of their diverse forms (such as the use of ghostwritten papers, the use of substitutes for taking examinations, the use of illegal cribs, plagiarism, and copying during examinations) is dishonest and must not be tolerated. Moreover, knowingly to aid and abet, directly or indirectly, other parties in committing dishonest acts is in itself dishonest.” Incidents of academic misconduct in this course will be addressed by the course instructor and referred to the Office of Student Rights and Responsibilities (OSRR) for review at the university level. Any violation of course policies as it relates to academic integrity will result minimally in a failing or zero grade for that particular assignment or test, and at the instructors discretion may result in a failing grade for the course. In addition, all incidents of academic misconduct will be forwarded to OSRR, where university penalties, including removal from the university, may be considered.

**Boilermaker Honor Pledge:** “As a Boilermaker pursuing academic excellence, I pledge to be honest and true in all that I do. Accountable together – we are Purdue.”

https://www.purdue.edu/odos/osrr/honor-pledge/about.html.

**Nondiscrimination:** Purdue University is committed to maintaining a community which recognizes and values the inherent worth and dignity of every person; fosters tolerance, sensitivity, understanding, and mutual respect among its members; and encourages each individual to strive to reach his or her own potential. In pursuit of its goal of academic excellence, the University seeks to develop and nurture diversity. The University believes that diversity among its many members strengthens the institution, stimulates creativity, promotes the exchange of ideas, and enriches campus life. Purdue’s nondiscrimination policy can be found at


**Academic Accommodation of Students with Disabilities:** Purdue University strives to make learning experiences as accessible as possible. If you anticipate or experience physical or academic barriers based on disability, you are encouraged to contact the Disability Resource Center at: drc@purdue.edu or by phone: 765-494-1247.

In this mathematics course accommodations are managed between the instructor, student and DRC Testing Center. Students should see instructors outside class hours before or after class or during office hours – to share your Accommodation Memorandum for the current semester and discuss your accommodations as soon as possible.

**Mental Health:** If you find yourself beginning to feel some stress, anxiety, and/or feeling slightly overwhelmed, try WellTrack at https://purdue.welltrack.com/. Sign in
and find information and tools at your fingertips, available to you at any time. If you need support and information about options and resources, please see the Office of the Dean of Students, http://www.purdue.edu/odos, for drop-in hours (M-F, 8 am-5 pm). If you’re struggling and need mental health services: Purdue University is committed to advancing the mental health and well-being of its students. If you or someone you know is feeling overwhelmed, depressed, and/or in need of mental health support, services are available. For help and to speak with a clinician, contact Counseling and Psychological Services (CAPS) at 765-494-6995 or by going to CAPS office on the second floor of the Purdue University Student Health Center (PUSH). For urgent situations after hours, on weekends and holidays, call 765-494-6995 to speak with a clinician. Please see http://www.purdue.edu/caps/ for further information.

**Commercial Note Taking in Classes:** Notes taken in class are generally considered to be derivative works of the instructors presentations and materials, and they are thus subject to the instructors copyright in such presentations and materials. No individual is permitted to sell or otherwise barter notes, either to other students or to any commercial concern, for a course without the express written permission of the course instructor. See the Regulations on Student Conduct: Miscellaneous Conduct Regulations:

http://catalog.purdue.edu/content.php?catoid=8&navoid=8208#miscellaneous-conductregulations

**Major Campus Emergency:** In the event of a major campus emergency, course requirements, deadlines, and grading are subject to change that may be necessitated by a revised calendar or other circumstances beyond the instructor’s control. Relevant changes to this course will be posted on course websites.
EMERGENCY PREPAREDNESS LECTURE

As we begin this semester I want to take a few minutes and discuss emergency preparedness. Purdue University is a very safe campus and there is a low probability that a serious incident will occur here at Purdue. However, just as we receive a “safety briefing” each time we get on an aircraft, we want to emphasize our emergency procedures for evacuation and shelter in place incidents. Our preparedness will be critical IF an unexpected event occurs!

Emergency preparedness is your personal responsibility. Purdue University is actively preparing for natural disasters or human-caused incidents with the ultimate goal of maintaining a safe and secure campus. Let’s review the following procedures:

- For any emergency text or call 911.
- There are more than 300 Emergency Telephones (aka blue lights) throughout campus that connect directly to the Purdue Police Department (PUPD). If you feel threatened or need help, push the button and you will be connected right away.
- If we hear a fire alarm we will immediately evacuate the building and proceed to the sidewalk area near Class of 50 and the Recitation buildings. DO NOT remain underneath the breezeway of MATH.
  - Do not use the elevator.
- If we are notified of a Shelter in Place requirement for a tornado warning we will stop classroom or research activities and shelter in the lowest level of this building away from windows and doors. Our preferred location is the Basement of MATH (i.e. the lower basement level).
- If we are notified of a Shelter in Place requirement for a hazardous materials release we will shelter in our classroom shutting any open doors and windows.
- If we are notified of a Shelter in Place requirement for an active threat such as a shooting we will shelter in a room that is securable preferably without windows. Our preferred location is MATH 215.

Attached to the syllabus is an “Emergency Preparedness for Classrooms” sheet that provides additional preparedness information. Please review the sheet and the Emergency Preparedness website for additional emergency preparedness information.
EMERGENCY PREPAREDNESS SYLLABUS ATTACHMENT

EMERGENCY NOTIFICATION PROCEDURES are based on a simple concept – if you hear a fire alarm inside, proceed outside. If you hear a siren outside, proceed inside.

- **Indoor Fire Alarms** mean to stop class or research and **immediately evacuate** the building.
  - Proceed to your Emergency Assembly Area away from building doors. **Remain outside** until police, fire, or other emergency response personnel provide additional guidance or tell you it is safe to leave.
- **All Hazards Outdoor Emergency Warning Sirens** mean to **immediately** seek shelter (**Shelter in Place**) in a safe location within the closest building.
  - “Shelter in place” means seeking immediate shelter inside a building or University residence. This course of action may need to be taken during a tornado, an active threat including a shooting or a release of hazardous materials in the outside air. Once safely inside, find out more details about the emergency*. **Remain in place** until police, fire, or other emergency response personnel provide additional guidance or tell you it is safe to leave.

*In both cases, you should seek additional clarifying information by all means possible...Purdue Emergency Status page, text message, Twitter, Desktop Alert, Albertus Beacon, digital signs, email alert, TV, radio, etc....review the Purdue Emergency Warning Notification System multi-communication layers at [http://www.purdue.edu/ehps/emergency_preparedness/warning-system.html](http://www.purdue.edu/ehps/emergency_preparedness/warning-system.html)

EMERGENCY RESPONSE PROCEDURES:

- Review the **Emergency Procedures Guidelines** [https://www.purdue.edu/emergency_preparedness/flipchart/index.html](https://www.purdue.edu/emergency_preparedness/flipchart/index.html)
- Review the **Building Emergency Plan** (available on the Emergency Preparedness website or from the building deputy) for:
  - evacuation routes, exit points, and emergency assembly area
  - when and how to evacuate the building.
  - shelter in place procedures and locations
  - additional building specific procedures and requirements.

EMERGENCY PREPAREDNESS AWARENESS VIDEOS

- "Run. Hide. Fight."® is a 6-minute active shooter awareness video that illustrates what to look for and how to prepare and react to this type of incident. See: [https://www.youtube.com/watch?v=5mzI_Saj4Vs](https://www.youtube.com/watch?v=5mzI_Saj4Vs) (Link is also located on the EP website)

MORE INFORMATION
Reference the Emergency Preparedness web site for additional information: [https://www.purdue.edu/ehps/emergency_preparedness/](https://www.purdue.edu/ehps/emergency_preparedness/)