This description of modified arrangements will be updated and will likely need to be adjusted as the situation develops–current version 22 March 2020.

Following the announcement by Purdue University that classes will move online from 23 March 2020, we will move to online delivery of both lectures and office hours. As we discussed in class, we all have a role to play through our personal decisions in reducing the probability of transmission of COVID-19, and minimising face-to-face meetings with other individuals reduces the probability of transmission very slightly with each meeting we avoid. Cumulatively, this makes a significant difference, and thereby we may hope that individuals with additional health risk factors – immune-compromised, diabetics, older friends, colleagues and family members – are all more likely to endure through this public health crisis. I deeply appreciate all of your contributions towards this goal.

Here are the important points of which to take note as we switch to online delivery of the course:

(a) Nobody needs to feel stressed or concerned that the new arrangements will disadvantage them so far as final grades are concerned. Please do contact me if this remains a concern after reading about the new arrangements. All instructors at Purdue are aware that the times are exceptional, and that you as students will face many challenges in the coming weeks. We are here to support your studies and we will seek to assist you to achieve a good outcome from this semester.

(b) The course web-page detailed below remains the go-to resource for all information concerning the course.

(c) In MA 59800AHA, assessment will remain based on the six problem sets that will have been distributed on the course website by the end of the semester. Four of these assignments have already been put online, and the remaining two will go online in the coming weeks. A satisfactory grade will be gained through demonstration of appropriate engagement with the problem sets, in just the same way as I have described earlier in the course. Do contact me if you have any concerns.

(d) At present, the default plan is that Homeworks 4, 5 and 6 can be returned to me via email at twooley@purdue.edu. If I can switch this to Blackboard submission, I will, and I will advise accordingly. Some of you have been texting your homeworks, and so can send a pdf. I would advise that for hand-written solutions, you scan them with your smartphone or via computer, and send me the scanned images. Again, do contact me if this will pose a problem. I will return a pdf image of your homework to you expeditiously via email with annotations as usual, to assist with your learning experience. At present, it looks as
if the Purdue policies associated with FERPA compliance will prevent me from sending you numerical scores associated with these homeworks. I am currently investigating how this might be made possible via Blackboard or some other means.

(e) The content that we will cover will remain unaltered.

(f) Following our trial with Purdue Webex and many hours of other testing, I have concluded that our most promising looking plan is for lectures to continue on Tuesdays and Thursdays 10:30 - 11:45 via Zoom. To get hooked up with Zoom (if you have not already done this for other reasons), go to

https://zoom.us

You should be able to go to the upper right hand side of your screen to JOIN A MEETING, and then you can enter the Meeting ID

< See your emails for Meeting ID number >

which should take you through to the meeting space. I have set up your microphones to be on mute initially – you can switch yourselves to unmute. It looks as if feedback loops are quite easy to set up, so be aware that muting may be required to stabilise the sound from time to time.

Zoom has the option to record sessions, and I plan to do this. This will offer the option of uploading our Zoom session classes onto the Media site of Blackboard to be available to you all. This may be particularly useful if you are unable to get into Zoom during the scheduled class time. **Warning:** If you want your voice to remain unrecorded, then the way to ensure this is to keep your mic on mute throughout (you can send a text to ask a question if your prefer). The only people who will be able to see this recording are those in the class, so I hope that this will not be a concern for anyone – but definitely let me know if it is indeed a concern!

One issue with Zoom is that the resolution of the real-time session is not particularly high. It is higher than Purdue Webex, and I will be using a thicker pen to do better than our test session. However, the recorded session has significantly higher resolution than the real-time session, so do check out the Blackboard available recording if the low resolution causes difficulties. I should be able to get this uploaded to the Blackboard media site within an hour of the end of class, if all goes to plan.

As a back-up to the live and downloadable versions of the lectures, I will continue to produce the printed notes, and will endeavour to have these updated more or less in real time. These will continue to be available from the class website.

(g) At present, the default plan is that office hours will continue Tu 14:00-15:00, W 13:00-14:00, Th 14:30-15:30, but will switch from face-to-face to online meetings. I will again make use of Zoom (see the above address) for these meetings. Simply check-in to Zoom using the above Meeting ID when you want to show up – I will be inside the Zoom environment during the office hour periods – and we will see how this goes. I will attempt to maintain a Piazza interactive forum going in real-time during these office hours, and available also outside these times, but I am unclear how viable or useful this will be, since I have no experience and limited information concerning Piazza. See your emails for Piazza coordinates.
A reminder of the previous guidance (with slight amendments) for the course:

**Class Meeting Times:** Tuesdays and Thursdays 10:30 - 11:45 via Zoom (see above).
**Credit Hours:** 3 hours
**Course web page:**
https://www.math.purdue.edu/~twooley/2020aha/2020aha.html

**Prerequisites:** Elementary number theory and basic analysis.

**Instructor:** Prof. Trevor Wooley, twooley@purdue.edu
**Location:** 422 Math, Tel. 765-496-6439
**Office Hours:** Tu 14:00-15:00, W 13:00-14:00, Th 14:30-15:30 via Zoom

**Course Description:** This course serves as an introduction to the (Hardy- Littlewood) circle method, an important theme in analytic number theory, and complements the course MA59800 Section 76 by Tess Anderson on discrete analysis – taking them in parallel, an option that is encouraged but not required, would enhance the level of understanding of each of them. Background results from number theory and harmonic analysis will be reviewed as needed.

The (Hardy-Littlewood) circle method applies Fourier analysis to count rational or integral solutions of an equation or inequality in a manner respecting the inherent arithmetic. Developments in recent years have broadened its impact into additive combinatorics and discrete harmonic analysis beyond its more traditional role in quantitative arithmetic geometry. A highlight from the past five years is the full resolution of the Main Conjecture in Vinogradov’s mean value theorem.

We shall take as our central example Waring’s problem – the problem of understanding the number of representations of an integer as the sum of a fixed number of k-th powers of positive integers. Our aims are twofold: (i) to understand the scope and limitations of the circle method, and (ii) to gain some facility to apply the method, so from time to time there will be technical material that we’ll just cite rather than prove in any detail. This course is intended to be accessible to those without any background in analytic number theory.

**Course Content:**
(i) Discussion of Weyl’s inequality, Hua’s Lemma, and the simplest treatment of Waring’s problem. This provides an opportunity to discuss the key elements of the major arc analysis, that is, the singular integral and singular series, that together constitute the product of local densities. Density of integral zeros of diagonal equations.
(ii) Refinements to the major arc analysis, including use of Poisson summation. Sketch of Kloosterman method. Diminishing ranges. Diophantine equations arising as sums of binary forms.
(iii) Vinogradov’s methods, especially Vinogradov’s mean value theorem and ensuing analogue’s of Weyl’s inequality. Application to Waring’s problem and diagonal equations. The Main Conjecture and its consequences.
(v) Unrepresentation theory – the theory of exceptional sets of integers that fail to be represented in a specified form.
Learning outcomes: Students completing the course will: (i) acquire basic skills in an important theme of analytic number theory; (ii) gain experience applying analytic methods, and in particular harmonic analysis, to solve Diophantine problems; and (iii) be equipped to estimate exponential sums via the principal basic methods in the subject.

Course texts: The course will be based on the instructor’s lecture notes. Good texts for background reading and support are:


(b) H. Davenport, Analytic methods for Diophantine equations and Diophantine inequalities, Ann Arbor Publishers, Ann Arbor, 1962 or the LaTeXed version published by Cambridge University Press in 2005 [Friendlier for the basics, with material on general homogeneous cubics, but misses modern developments.]

(c) M. Nathanson, Additive number theory. The classical bases, GTM 164, Springer-Verlag, New York, 1996 [Pedestrian approach to the basics in which no corner is cut – good for getting started!]

Assessment: Course credit will be based solely on six (short) problem sets offered through the semester, posted on the course web-page:

https://www.math.purdue.edu/~twooley/2020aha/2020aha.html

Class participants can demonstrate engagement with the course by any written and/or in-class presentations featuring a reasonable subset of these problems. There are three levels of difficulty: short problems testing basic skill-sets, extended problems integrating the essential methods of the course, and more challenging problems for enthusiasts with detailed hints available on request. Working with other class members is permitted, but do write up the solutions individually by yourselves.

Boilerplate Notes for Boilermakers:

Attendance: In the event of an absence for medical or other reasons, students should review and follow the Student Regulations concerning Classes, informing the instructor as far in advance as possible.

https://www.purdue.edu/studentregulations/regulations_procedures/classes.html

Academic Integrity: Academic integrity is one of the highest values that Purdue University holds. Individuals are encouraged to alert university officials to potential breaches of this value by either emailing integrity@purdue.edu, or by calling 765-494-8778, or by contacting the Office of the Dean of Students (https://www.purdue.edu/odos/). While information may be submitted anonymously, the more information that is submitted provides the greatest opportunity for the university to investigate the concern. Purdue prohibits “dishonesty in connection with any University activity. Cheating, plagiarism, or knowingly furnishing false information to the University are examples of dishonesty” (Section B.2.a of the Student Regulations

https://www.purdue.edu/studentregulations/student_conduct/regulations.html).
Furthermore, the University Senate has stipulated that “the commitment of acts of cheating, lying, and deceit in any of their diverse forms (such as the use of ghostwritten papers, the use of substitutes for taking examinations, the use of illegal cribs, plagiarism, and copying during examinations) is dishonest and must not be tolerated. Moreover, knowingly to aid and abet, directly or indirectly, other parties in committing dishonest acts is in itself dishonest.” Incidents of academic misconduct in this course will be addressed by the course instructor and referred to the Office of Student Rights and Responsibilities (OSRR) for review at the university level. Any violation of course policies as it relates to academic integrity will result minimally in a failing or zero grade for that particular assignment or test, and at the instructors discretion may result in a failing grade for the course. In addition, all incidents of academic misconduct will be forwarded to OSRR, where university penalties, including removal from the university, may be considered.

Boilermaker Honor Pledge: “As a Boilermaker pursuing academic excellence, I pledge to be honest and true in all that I do. Accountable together – we are Purdue.”

https://www.purdue.edu/odos/osrr/honor-pledge/about.html.

Nondiscrimination: Purdue University is committed to maintaining a community which recognizes and values the inherent worth and dignity of every person; fosters tolerance, sensitivity, understanding, and mutual respect among its members; and encourages each individual to strive to reach his or her own potential. In pursuit of its goal of academic excellence, the University seeks to develop and nurture diversity. The University believes that diversity among its many members strengthens the institution, stimulates creativity, promotes the exchange of ideas, and enriches campus life. Purdue’s nondiscrimination policy can be found at


Academic Accommodation of Students with Disabilities: Purdue University strives to make learning experiences as accessible as possible. If you anticipate or experience physical or academic barriers based on disability, you are encouraged to contact the Disability Resource Center at: drc@purdue.edu or by phone: 765-494-1247.

In this mathematics course accommodations are managed between the instructor, student and DRC Testing Center. Students should see instructors outside class hours before or after class or during office hours – to share your Accommodation Memorandum for the current semester and discuss your accommodations as soon as possible.

Mental Health: If you find yourself beginning to feel some stress, anxiety, and/or feeling slightly overwhelmed, try WellTrack at https://purdue.welltrack.com/. Sign in and find information and tools at your fingertips, available to you at any time. If you need support and information about options and resources, please see the Office of the Dean of Students, http://www.purdue.edu/odos, for drop-in hours (M-F, 8 am-5 pm). If you’re struggling and need mental health services: Purdue University is committed to advancing the mental health and well-being of its students. If you or someone you know is feeling overwhelmed, depressed, and/or in need of mental health support, services are available. For help and to speak with a clinician, contact Counseling and Psychological Services (CAPS) at 765-4946995 or by going to CAPS office on the second floor of the Purdue University Student Health Center (PUSH). For urgent situations after
hours, on weekends and holidays, call 765-494-6995 to speak with a clinician. Please see http://www.purdue.edu/caps/ for further information.

**Commercial Note Taking in Classes:** Notes taken in class are generally considered to be derivative works of the instructors presentations and materials, and they are thus subject to the instructors copyright in such presentations and materials. No individual is permitted to sell or otherwise barter notes, either to other students or to any commercial concern, for a course without the express written permission of the course instructor. See the Regulations on Student Conduct: Miscellaneous Conduct Regulations:

http://catalog.purdue.edu/content.php?catoid=8&navoid=8208#miscellaneous-conductregulations

**Major Campus Emergency:** In the event of a major campus emergency, course requirements, deadlines, and grading are subject to change that may be necessitated by a revised calendar or other circumstances beyond the instructor’s control. Relevant changes to this course will be posted on course websites.