MA 59800AANT SECTION 084, CRN 29313
ANALYTIC NUMBER THEORY, II: A SECOND COURSE

TREVOR D. WOOLEY

Class Meeting Times: Mondays, Wednesdays and Fridays 10:30 - 11:20
Class location/modality: Face-to-face in REC 225.
Credit Hours: 3 hours
Course web page:
https://www.math.purdue.edu/∼twooley/2021aant2/2021aant2.html
Course Brightspace page:
https://purdue.brightspace.com/d2l/home/216055
Prerequisites: A first course in analytic number theory, basic real and complex analysis.

Instructor: Prof. Trevor Wooley, twooley@purdue.edu
Location: 422 Math, Tel. 765-496-6439
Office Hours: M 15:30-16:30, W 16:00-17:00, F 13:30-14:30 via Zoom

Course Description: This course serves as a sequel to the first course in analytic number theory taught in Fall 2020. It will serve as a gateway to advanced topics interfacing with problems active in current research concerning the analytic theory of numbers. As such it explores the finer aspects of the distribution of prime numbers in arithmetic progression and consequences for such problems as the Goldbach and Twin Prime problems. Along the way, one encounters the distribution of zeros of the Riemann zeta-function and Dirichlet L-functions, and important estimates for exponential and character sums. Following Dirichlet's proof in 1837 of the infinitude of primes in arithmetic progressions b modulo q (with b and q coprime), and the proof of the Prime Number Theorem by Hadamard and de la Vallee Poussin in 1896, researchers turned to examine finer questions concerning the distribution of prime numbers. How small is the smallest prime number congruent to b modulo q? What can be said if one averages over the modulus q? How are such results connected with the zeros of Dirichlet L-functions? How close can one come to establishing the Riemann Hypothesis for such functions, and what would this imply about the distribution of primes? Do any of these results lead to interesting implications for the famous conjectures about primes? This is a second course in analytic number theory that explores important methods and estimates that lay the foundation for research in the modern theory. Students interested in broadening their knowledge of analytic methods relevant in harmonic analysis and analytic number theory will find much of this course useful, as will those preparing for research in the area ... and there are many beautiful results and theoretical developments along the way to maintain the interest of enthusiasts.

Course Content:
Basic Topics. (i) The large sieve for exponential and character sums; (ii) Exponential sums over primes; (iii) The Goldbach problem: sums of two and three primes; (iv) The Bombieri-Vinogradov theorem on primes in arithmetic progression; (v) The Barban-Davenport-Halberstam theorem on the variance of primes in arithmetic progression; (vi) Exponential sum estimates via van der Corput's methods; (vii) The Burgess estimate for character sums in short intervals; (viii) Mean and large values of Dirichlet series; (ix) Approximate functional equations; (x) Bounds for the Riemann zeta function and L-functions in the critical strip; (xi) Exponential sum estimates via Vinogradov's methods; (xii) The zero-free region for the Riemann zeta function and refined asymptotics in the prime number theorem; (xiii) Zero density estimates; (xiv) Primes in short intervals; (xv) The Deuring-Heilbronn phenomenon; (xvi) Linnik's theorem on the smallest prime in an arithmetic progression.
Advanced topics (depending on demand and available time) may include an introduction to sieves, Maynard's theorem on prime k-tuples, pair correlation and zero density of the zeros of the Riemann zeta function.

Learning outcomes: Students completing the course will: (i) acquire basic skills in an important theme of analytic number theory; (ii) gain experience applying analytic methods to solve problems in multiplicative number theory; and (iii) be equipped to apply Dirichlet series and character sums via the principal basic methods in the subject.

Course texts: The course will be based on the instructors lecture notes, which in turn are based on:

(a) H. Davenport, *Multiplicative Number Theory*, 2nd ed., Springer-Verlag, GTM 74, 1980;

(b) H. Iwaniec and E. Kowalski, *Analytic Number Theory*, AMS Colloquium Series Vol. 53, 2004;

(c) H. L. Montgomery and R. C. Vaughan, *Multiplicative Number Theory, I. Classical Theory*, Cambridge Studies in Advanced Mathematics 97, Cambridge University Press, 2007 (and the unpublished volume II).

Assessment: Course credit will be based solely on six (short) problem sets offered once every 2 weeks through the semester, posted on the course web-page:

https://www.math.purdue.edu/∼twooley/2021aant2/2021aant2.html

Each homework is worth an equal amount of credit. Class participants can demonstrate engagement with the course by any written and/or in-class presentations featuring a reasonable subset of these problems. There are three levels of difficulty: short problems testing basic skill-sets, extended problems integrating the essential methods of the course, and more challenging problems for enthusiasts with detailed hints available on request. Working with other class members is permitted, but do write up the solutions individually by yourselves.

Purdue mandates that I include the following:
"Students who get at least 97% of the total points in this course are guaranteed an A+, 93% guarantees an A, 90% an A-, 87% a B+, 83% a B, 80% a B-, 77% a C+, 73% a

C, 70% a C-, 67% a D+, 63% a D, and 60% a D-; for each of these grades, it's possible that at the end of the semester a somewhat lower percentage will be enough to get that grade".

This is a graduate course, and you should imagine that a more nuanced approach will be taken in determining grades that reflects the demonstration of engagement outlined above. In particular, it is likely that the numerical scores required to achieve the grades listed will be lower than those indicated in the mandated text.

Arrangements for online aspects: We will use Zoom for Office Hours. This should be available at:

https://zoom.us

Go to the upper right hand side of your screen to JOIN A MEETING, and then enter the

Meeting ID: xxx-xxx-xxxx
Password: xxxxx (***For anyone from 598 AHA in SPRING, note that this is a different PIN !!!***)

This should take you through to the meeting space. I believe that you should be able to use the following short-cut, but if that does not work, do please use the Meeting ID and Password!

https://purdue-edu.zoom.us/j/xxxxxxxxxx?pwd=xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx

I will update my own notes as a back-up to the in-class lectures, more or less in real time. These will be available from the class website.

Some tips on Zoom office-hours:
(1) If you do not have the Zoom app on your computer, then do consider downloading this in advance. It's free!
(2) If we run into technical problems, be patient and attempt to re-enter the Zoom meeting room. I have found Zoom to be pretty stable, but if anything freezes or drops out, I'll attempt to restart and with luck, we will lose only a couple of minutes.
(3) If technology fails at your end – don't panic!

Homework submission and return: We will use Brightspace/Gradescope for homework submission, return and as a portal for you to receive grades and feedback on your homeworks. If you run into problems with homework submission, please just drop me an email at twooley@purdue.edu and we will find a workaround. The expectation is that pdfs of homeworks can be submitted directly, and that hand-written solutions can be scanned with a smartphone or via computer, and the scanned images submitted.

Office hours: The default plan is to hold these via Zoom as for the classes (see the above guidance). Simply check-in to Zoom when you want to show up – I will be inside the Zoom environment during the office hour periods.

Boilerplate Notes for Boilermakers:

Academic guidance in the event you are quarantined/isolated. If you become quarantined or isolated at any point in time during the semester, in addition to support from the Protect Purdue Health Center, you will also have access to an Academic Case Manager who can provide you academic support during this time. Your Academic Case Manager can be reached at acmq@purdue.edu and will provide you with general guidelines/resources around communicating with your instructors, be available for academic support, and offer suggestions for how to be successful when learning remotely. Importantly, if you find yourself too sick to progress in the course, notify your academic case manager and notify me via email. We will make arrangements based on your particular situation. The Office of the Dean of Students (odos@purdue.edu) is also available to support you should this situation occur.

Attendance: Students are expected to attend all classes in-person unless they are ill or otherwise unable to attend class. If they feel ill, have any symptoms associated with COVID-19, or suspect they have been exposed to the virus, students should stay home and contact the Protect Purdue Health Center (496-INFO).

In the current context of COVID-19, in-person attendance cannot be a factor in the final grades. However, timely completion of alternative assessments can certainly be part of the final grade. Students need to inform the instructor of any conflict that can be anticipated and will affect the timely submission of an assignment or the ability to take an exam.

Classroom engagement is extremely important and associated with your overall success in the course. The importance and value of course engagement and ways in which you can engage with the course content even if you are in quarantine or isolation, will be discussed at the beginning of the semester. Student survey data from Fall 2020 emphasized students views of in-person course opportunities as critical to their learning, engagement with faculty/TAs, and ability to interact with peers.

Only the instructor can excuse a student from a course requirement or responsibility. When conflicts can be anticipated, such as for many University-sponsored activities and religious observations, the student should inform the instructor of the situation as far in advance as possible. For unanticipated or emergency conflicts, when advance notification to an instructor is not possible, the student should contact the instructor/instructional team as soon as possible by email, through Brightspace, or by phone. In cases of bereavement, quarantine, or isolation, the student or the students representative should contact the Office of the Dean of Students via email or phone at 765-494-1747. Our course Brightspace includes a link to the Dean of Students under Campus Resources.

If you must quarantine or isolate at any point in time during the semester, please reach out to me via email so that we can communicate about how you can continue to learn remotely. Work with the Protect Purdue Health Center (PPHC) to get documentation and support, including access to an Academic Case Manager who can provide you with general guidelines/resources around communicating with your instructors, be available for academic support, and offer suggestions for how to be successful when learning remotely. Your Academic Case Manager can be reached at acmq@purdue.edu. Importantly, if you find yourself too sick to progress in the course, notify your academic case manager and notify me via email. We will make arrangements based on your particular situation.

Protect Purdue: The Protect Purdue Plan, which includes the Protect Purdue Pledge, is campus policy and as such all members of the Purdue community must comply with the required health and safety guidelines. Required behaviors in this class include: staying home and contacting the Protect Purdue Health Center (496-INFO) if you feel ill or know you have been exposed to the virus, properly wearing a mask in classrooms and campus building, at all times (e.g., mask covers nose and mouth, no eating/drinking in the classroom), disinfecting desk/workspace prior to and after use, maintaining appropriate social distancing with peers and instructors (including when entering/exiting classrooms), refraining from moving furniture, avoiding shared use of personal items, maintaining robust hygiene (e.g., handwashing, disposal of tissues) prior to, during and after class, and following all safety directions from the instructor.

Students who are not engaging in these behaviors (e.g., wearing a mask) will be offered the opportunity to comply. If non-compliance continues, possible results include instructors asking the student to leave class and instructors dismissing the whole class. Students who do not comply with the required health behaviors are violating the University Code of Conduct and will be reported to the Dean of Students Office with sanctions ranging from educational requirements to dismissal from the university.

Any student who has substantial reason to believe that another person in a campus room (e.g., classroom) is threatening the safety of others by not complying (e.g., not wearing a mask) may leave the room without consequence. The student is encouraged to report the behavior to and discuss next steps with their instructor. Students also have the option of reporting the behavior to the Office of the Student Rights and Responsibilities. See also Purdue University Bill of Student Rights.

Academic Integrity: Academic integrity is one of the highest values that Purdue University holds. Individuals are encouraged to alert university officials to potential breaches of this value by either emailing integrity@purdue.edu, or by calling 765-494-8778, or by contacting the Office of the Dean of Students (https://www.purdue.edu/odos/). While information may be submitted anonymously, the more information that is submitted provides the greatest opportunity for the university to investigate the concern. Purdue prohibits "dishonesty in connection with any University activity. Cheating, plagiarism, or knowingly furnishing false information to the University are examples of dishonesty" (Section B.2.a of the Student Regulations

https://www.purdue.edu/studentregulations/student_conduct/regulations.html).

Furthermore, the University Senate has stipulated that "the commitment of acts of cheating, lying, and deceit in any of their diverse forms (such as the use of ghostwritten papers, the use of substitutes for taking examinations, the use of illegal cribs, plagiarism, and copying during examinations) is dishonest and must not be tolerated. Moreover, knowingly to aid and abet, directly or indirectly, other parties in committing dishonest acts is in itself dishonest." Incidents of academic misconduct in this course will be addressed by the course instructor and referred to the Office of Student Rights and Responsibilities (OSRR) for review at the university level. Any violation of course policies as it relates to academic integrity will result minimally in a failing or zero grade for that particular assignment or test, and at the instructors discretion may result in a failing grade for the course. In addition, all incidents of academic misconduct will be forwarded to OSRR, where university penalties, including removal from the university, may be considered.

Boilermaker Honor Pledge: "As a Boilermaker pursuing academic excellence, I pledge to be honest and true in all that I do. Accountable together – we are Purdue."

https://www.purdue.edu/odos/osrr/honor-pledge/about.html.

Nondiscrimination: Purdue University is committed to maintaining a community which recognizes and values the inherent worth and dignity of every person; fosters tolerance, sensitivity, understanding, and mutual respect among its members; and encourages each individual to strive to reach his or her own potential. In pursuit of its goal of academic excellence, the University seeks to develop and nurture diversity. The University believes that diversity among its many members strengthens the institution, stimulates creativity, promotes the exchange of ideas, and enriches campus life. More details are available on our course Brightspace table of contents, under University Policies.

Purdue's nondiscrimination policy can be found at

https://www.purdue.edu/purdue/ea_eou_statement.php.

Academic Accommodation of Students with Disabilities: Purdue University strives to make learning experiences accessible to all participants. If you anticipate or experience physical or academic barriers based on disability, you are encouraged to contact the Disability Resource Center at: drc@purdue.edu or by phone: 765-494-1247.

In this mathematics course accommodations are managed between the instructor, the student and DRC Testing Center. If you have been certified by the Disability Resource Center (DRC) as eligible for accommodations, you should contact your instructor to discuss your accommodations as soon as possible. Here are instructions for sending your Course Accessibility Letter to your instructor:

https://www.purdue.edu/drc/students/course-accessibility-letter.php

Mental Health: If you find yourself beginning to feel some stress, anxiety, and/or feeling slightly overwhelmed, try WellTrack at https://purdue.welltrack.com/. Sign in and find information and tools at your fingertips, available to you at any time. If you need support and information about options and resources, please see the Office of the Dean of Students, http://www.purdue.edu/odos, for drop-in hours (M-F, 8 am-5 pm). If you find yourself struggling to find a healthy balance between academics, social life, stress, etc. sign up for free one-on-one virtual or in-person sessions with a Purdue Wellness Coach at RecWell. Student coaches can help you navigate through barriers and challenges toward your goals throughout the semester. Sign up is completely free and can be done on BoilerConnect. If you have any questions, please contact Purdue Wellness at evans240@purdue.edu.If you're struggling and need mental health services: Purdue University is committed to advancing the mental health and well-being of its students. If you or someone you know is feeling overwhelmed, depressed, and/or in need of mental health support, services are available. For help and to speak with a clinician, contact Counseling and Psychological Services (CAPS) at 765-494-6995 or by going to CAPS office on the second floor of the Purdue University Student Health Center (PUSH). For urgent situations after hours, on weekends and holidays, call 765-494-6995 to speak with a clinician. Please see http://www.purdue.edu/caps/ for further information.

Commercial Note Taking in Classes: Notes taken in class are generally considered to be derivative works of the instructors presentations and materials, and they are thus subject to the instructors copyright in such presentations and materials. No individual is

permitted to sell or otherwise barter notes, either to other students or to any commercial concern, for a course without the express written permission of the course instructor. See the Regulations on Student Conduct: Miscellaneous Conduct Regulations:

http://catalog.purdue.edu/content.php?catoid=8&navoid=8208#miscellaneous-conductregulations

Major Campus Emergency: In the event of a major campus emergency, course requirements, deadlines and grading percentages are subject to changes that may be necessitated by a revised semester calendar or other circumstances beyond the instructors control. Relevant changes to this course will be posted onto the course website or can be obtained by contacting the instructors or TAs via email or phone. You are expected to read your @purdue.edu email on a frequent basis.

EMERGENCY PREPAREDNESS LECTURE

As we begin this semester I want to take a few minutes and discuss emergency preparedness. Purdue University is a very safe campus and there is a low probability that a serious incident will occur here at Purdue. However, just as we receive a "safety briefing" each time we get on an aircraft, we want to emphasize our emergency procedures for evacuation and shelter in place incidents. Our preparedness will be critical IF an unexpected event occurs!

Emergency preparedness is your personal responsibility. Purdue University is actively preparing for natural disasters or human-caused incidents with the ultimate goal of maintaining a safe and secure campus. Let's review the following procedures:

- For any emergency text or call 911.
- There are more than 300 Emergency Telephones (aka blue lights) throughout campus that connect directly to the Purdue Police Department (PUPD). If you feel threatened or need help, push the button and you will be connected right away.
- If we hear a fire alarm we will immediately evacuate the building and proceed to the emergency assembly area near the fountain at John Purdue's grave on the Memorial Mall
    - **Do not use the elevator.**
- If we are notified of a Shelter in Place requirement for a tornado warning we will stop classroom or research activities and shelter in the lowest level of this building away from windows and doors.
- If we are notified of a Shelter in Place requirement for a hazardous materials release we will shelter in our classroom shutting any open doors and windows.
- If we are notified of a Shelter in Place requirement for an active threat such as a shooting we will shelter in a room that is securable without any windows. Our preferred location is REC 225.

Attached to the syllabus is an "Emergency Preparedness for Classrooms" sheet that provides additional preparedness information. Please review the sheet and the Emergency Preparedness website for additional emergency preparedness information.

EMERGENCY PREPAREDNESS SYLLABUS ATTACHMENT

EMERGENCY NOTIFICATION PROCEDURES are based on a simple concept – if you hear a fire alarm inside, proceed outside. If you hear a siren outside, proceed inside.

- **Indoor Fire Alarms** mean to stop class or research and immediately **evacuate** the building.
  - Proceed to your Emergency Assembly Area away from building doors. **Remain outside** until police, fire, or other emergency response personnel provide additional guidance or tell you it is safe to leave.
- **All Hazards Outdoor Emergency Warning Sirens** mean to immediately seek shelter (**Shelter in Place**) in a safe location within the closest building.
  - "Shelter in place" means seeking immediate shelter inside a building or University residence. This course of action may need to be taken during a tornado, an active threat including a shooting or a release of hazardous materials in the outside air. Once safely inside, find out more details about the emergency*. **Remain in place** until police, fire, or other emergency response personnel provide additional guidance or tell you it is safe to leave.

**In both cases, you should seek additional clarifying information by all means possible…Purdue Emergency Status page, text message, Twitter, Desktop Alert, Albertus Beacon, digital signs, email alert, TV, radio, etc.…review the Purdue Emergency Warning Notification System multi-communication layers at http://www.purdue.edu/ehps/emergency_preparedness/warning-system.html*

EMERGENCY RESPONSE PROCEDURES:

- Review the **Emergency Procedures Guidelines** https://www.purdue.edu/emergency_preparedness/flipchart/index.html
- Review the **Building Emergency Plan** (available on the Emergency Preparedness website or from the building deputy) for:
  - evacuation routes, exit points, and emergency assembly area
  - when and how to evacuate the building.
  - shelter in place procedures and locations
  - additional building specific procedures and requirements.

EMERGENCY PREPAREDNESS AWARENESS VIDEOS

- **"Run. Hide. Fight.®"** is a 6-minute active shooter awareness video that illustrates what to look for and how to prepare and react to this type of incident. See: https://www.youtube.com/watch?v=5mzI_5aj4Vs (Link is also located on the EP website)

MORE INFORMATION

Reference the Emergency Preparedness web site for additional information: https://www.purdue.edu/ehps/emergency_preparedness/