GALOIS THEORY: HOMEWORK 11

Due 6pm Wednesday 3rd April 2024

1. Suppose that $L : M : K$ is an algebraic tower of fields. Prove that $L : K$ is separable if and only if $L : M$ and $M : K$ are both separable. [Hint: try using the Primitive Element Theorem].
2. Suppose that $E : K$ and $F : K$ are finite extensions with $K \subseteq E \subseteq L$ and $K \subseteq F \subseteq L$, with L a field.
   (a) Show that when $E : K$ is separable, then so too is $EF : F$.
   (b) Show that when $E : K$ and $F : K$ are both separable, then so too are $EF : K$ and $E \cap F : K$.
3. Suppose that $\mathrm{char}(K) = p > 0$ and that $L : K$ is a totally inseparable algebraic extension (thus, every element of $L \setminus K$ is inseparable). Show that whenever $\alpha \in L$, then there is a non-negative integer n and an element $\theta \in K$ having the property that $m_\alpha(K) = t^{p^n} - \theta$.