GALOIS THEORY: HOMEWORK 5

Due 6pm Wednesday 14th February 2024

1. Suppose that $L : F$ and $L : F'$ are finite extensions with $F \subseteq L$ and $F' \subseteq L$, and further that $\psi : F \to F'$ is an isomorphism. Explain why there are at most $[L : F]$ ways to extend ψ to a homomorphism from L into L. [This is Corollary 3.6 – consider F-homomorphisms acting on L.]

2. Let M be a field. Show that the following are equivalent:
   (i) the field M is algebraically closed;
   (ii) every non-constant polynomial $f \in M[t]$ factors in $M[t]$ as a product of linear factors;
   (iii) every irreducible polynomial in $M[t]$ has degree 1;
   (iv) the only algebraic extension of M containing M is M itself.

3. Revise for the first mid-term!