NUMBER THEORY: HOMEWORK 5

TO BE HANDED IN BY THURSDAY 20TH FEBRUARY 2025 BY 6PM

1. (a) Find a solution of $x^4 + x + 4 \equiv 0 \pmod{3^3}$.

(b) Show that $x^2 + 4x + 18 \equiv 0 \pmod{49}$ has no solutions.

2. (a) Prove that if a belongs to h modulo a prime p (i.e. a has order h modulo p), and if h is even, then $a^{h/2} \equiv -1 \pmod{p}$.

(b) Suppose that p is odd and a belongs to h modulo p^k for some integer $k \ge 2$. Is it necessarily the case that $a^{h/2} \equiv -1 \pmod{p^k}$? Explain your answer.

3. Show that $x^p \equiv x \pmod{p^j}$ has precisely p solutions modulo p^j for every prime power p^j .

©Trevor D. Wooley, Purdue University 2025. This material is copyright of Trevor D. Wooley at Purdue University unless explicitly stated otherwise. It is provided exclusively for educational purposes at Purdue University, and is to be downloaded or copied for your private study only.