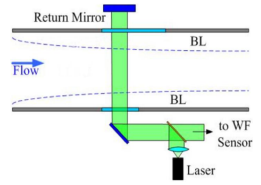
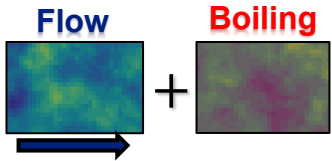


Boiling flow parameter estimation from boundary layer data



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²Departments of Electrical and Computer Engineering, and Biomedical Engineering, Purdue University

³Department of Engineering Physics, Air Force Institute of Technology



Light Detection and Transmission

Goal: *Detect and transmit* light from the side of a high-speed aircraft.



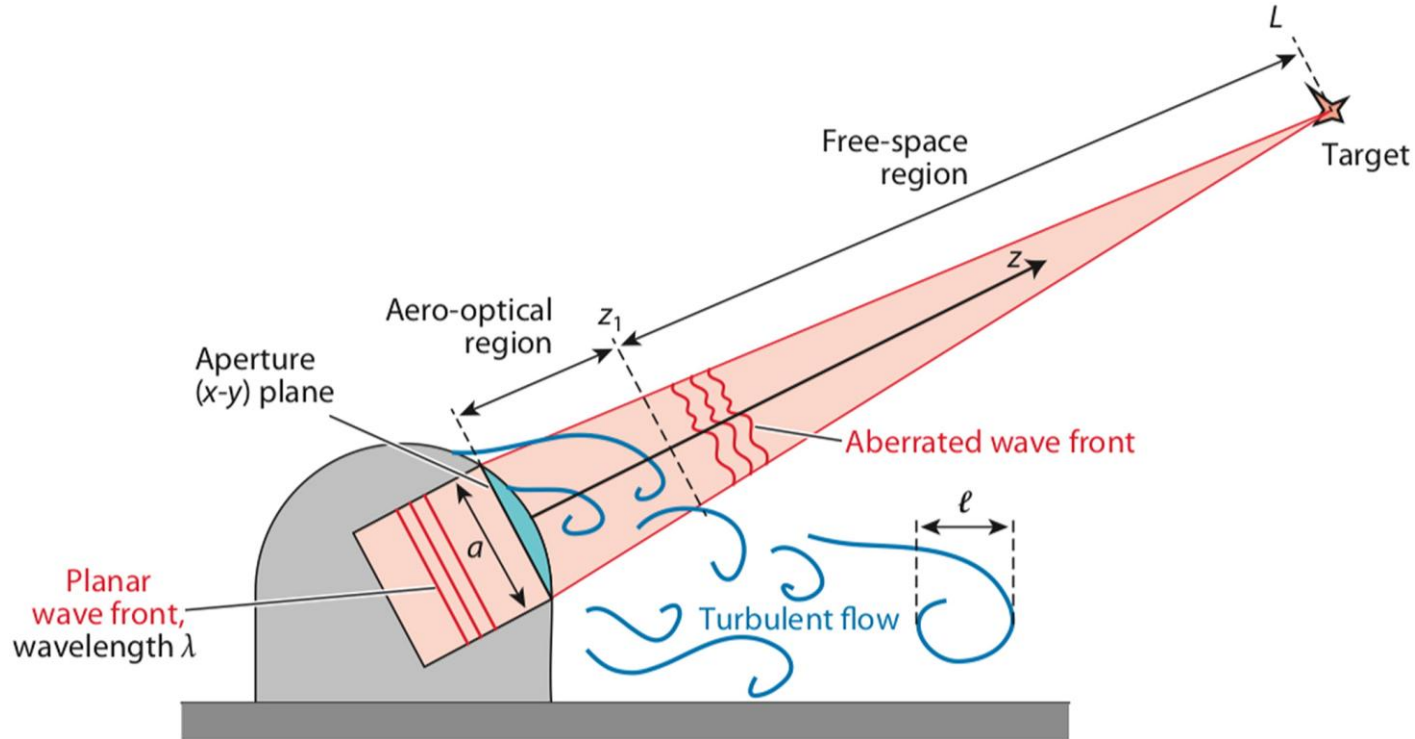
<https://aero-optics.nd.edu/research/hemispherical-turret-beam-directors/>



[1] E. J. Jumper, S. Gordeyev, and M. R. Whiteley, "Aero-optical effects," in *Aero-Optical Effects*, (John Wiley Sons, Incorporated, United States, 2023).

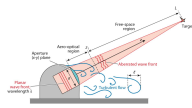
The Aero-Optics Problem

Problem: Turbulent flow near the aperture **distorts light propagation.**

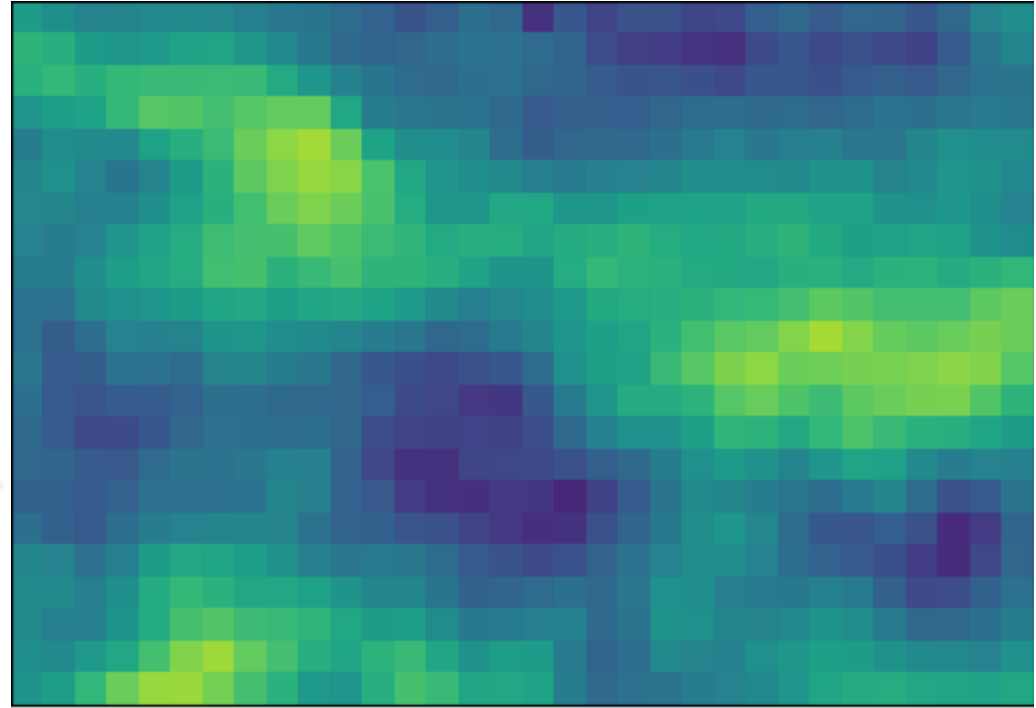


[2] M. Wang, A. Mani, and S. Gordeyev, "Physics and Computation of Aero-Optics," *Annual Review of Fluid Mechanics*, Vol. 44, No. 1, 2012, pp. 299–321.

Phase Screen Generation for Aero-Optic Effects



➤ We quantify
aero-optic effects
as *time-series of*
phase screens.

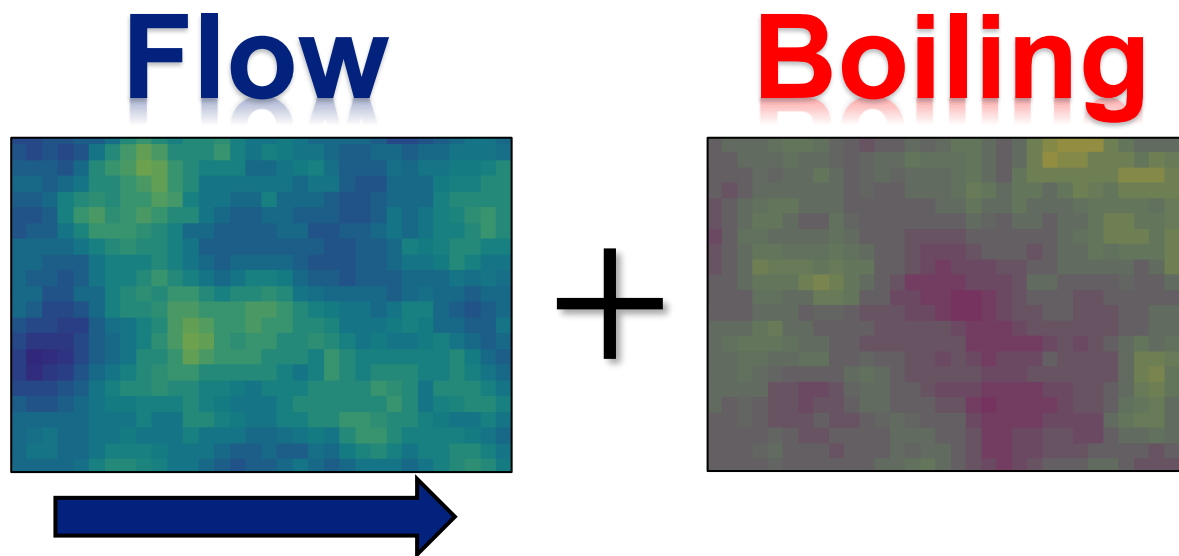


➤ Phase screens can be *measured* by experiment
(**expensive**) or *simulated* (**less expensive**).

Phase Screen Simulation: Boiling Flow

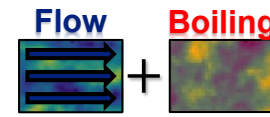
Boiling Flow adds two components at each time-step:

1. **Flow**: *Shift* the previous phase screen according to some *flow velocity*.
2. **Boiling**: A *new* random Kolmogorov phase screen.



[3] Srinath, S., Poyneer, L. A., Rudy, A. R., and Ammons, S. M., "Computationally efficient autoregressive method for generating phase screens with frozen flow and turbulence in optical simulations," Opt. Express 23, 33335–33349 (Dec 2015)

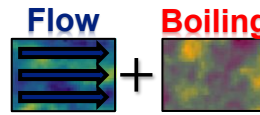
Phase Screen Simulation: Boiling Flow



$$\phi_n = \alpha \text{Flow}(\phi_{n-1}) + \sqrt{1 - \alpha^2} \text{Boiling}$$

A large diagram illustrating the phase screen simulation equation. It shows a large square on the left representing the result, followed by an equals sign, a green Greek letter alpha, a square labeled 'Flow' above it, a plus sign, a square root of 1 minus alpha squared, and a square labeled 'Boiling' above it. The 'Flow' square has a blue and green speckled pattern, and the 'Boiling' square has a dark purple and green speckled pattern.
$$\text{Result} = \alpha \text{Flow} + \sqrt{1 - \alpha^2} \text{Boiling}$$

Boiling Flow Parameters

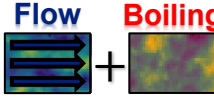


$$\phi_n = \alpha \mathbf{Flow}(\phi_{n-1}) + \sqrt{1 - \alpha^2} \mathbf{Boiling}$$

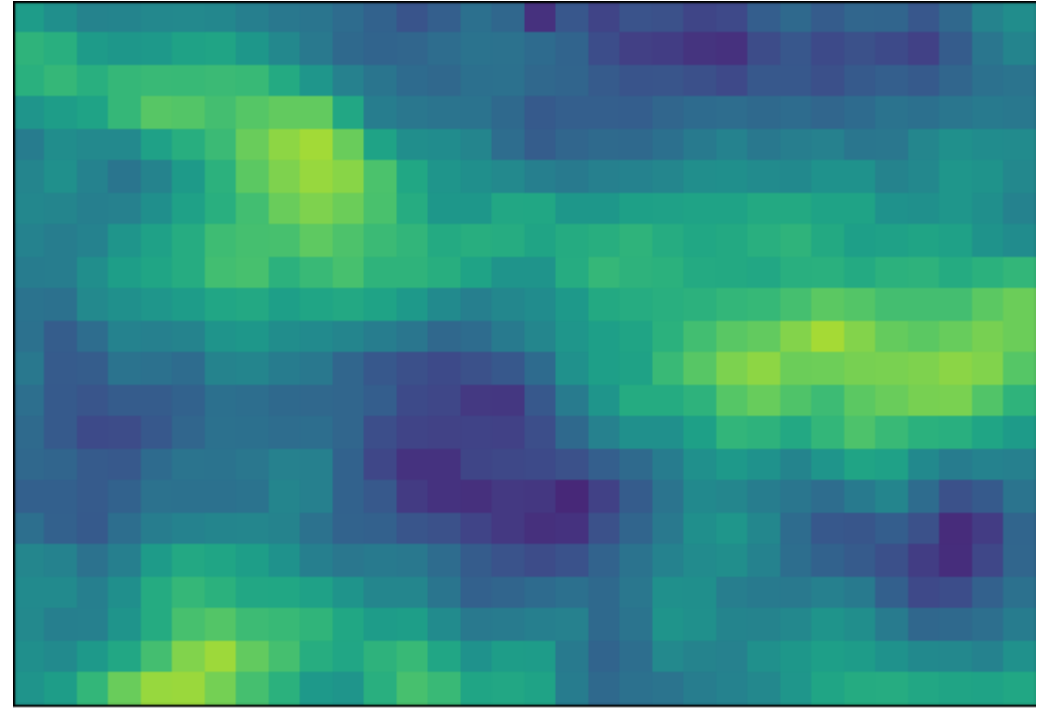
Parameters:

- L_0 : outer scale (m)
- r_0 : Fried coherence length (m)
- (v_x, v_y) : flow velocity components (pixels/time-step)
- α : *boiling coefficient* (unit-less)

Phase Screen Generation for Aero-Optic Effects

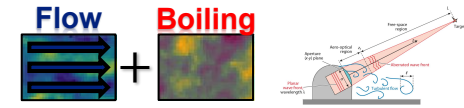


➤ **Problem**: Boiling flow depends on *physical parameters*, some of which are **undefined for aero-optic effects**.

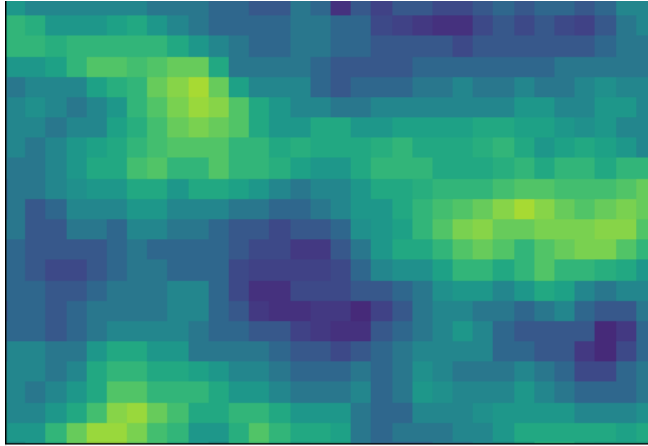


➤ **Solution**: We develop an algorithm to **estimate** these parameters **from measured aero-optic phase screens**.

Parameter Estimation from Measured Data



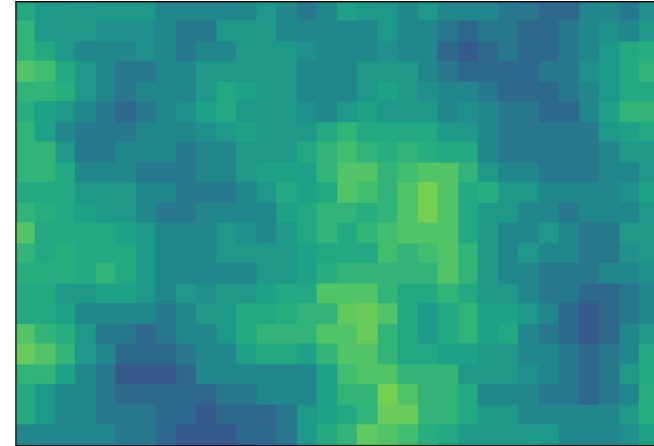
Measured Phase Screens



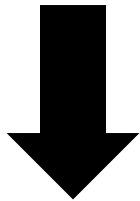
Compare



Synthetic Phase Screens



Estimate



$$\left(\hat{L}_0, \hat{r}_0, \hat{v}_x, \hat{v}_y, \hat{\alpha} \right)$$

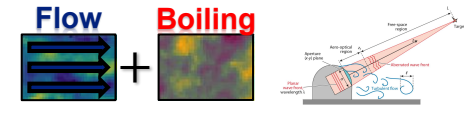


Generate

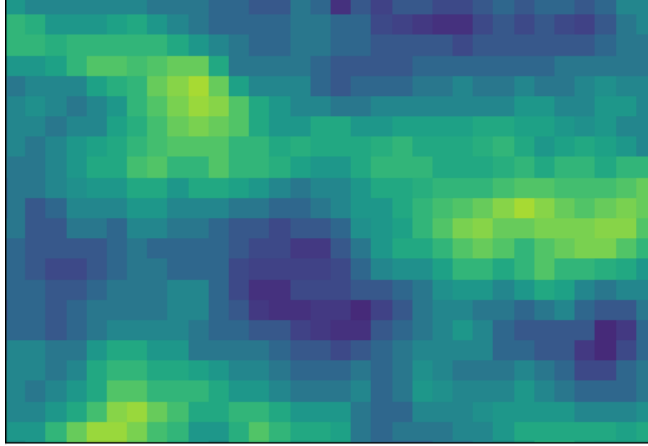


Boiling Flow

Parameter Estimation from Measured Data



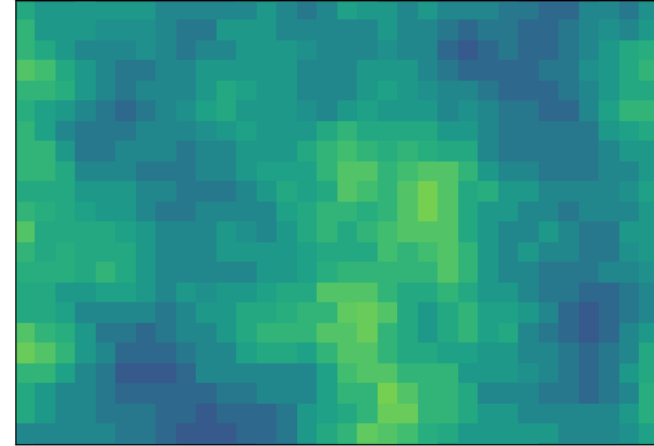
Measured Phase Screens



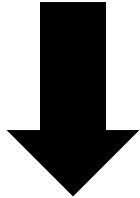
Compare



Synthetic Phase Screens

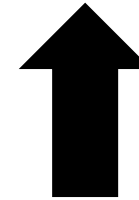


Estimate



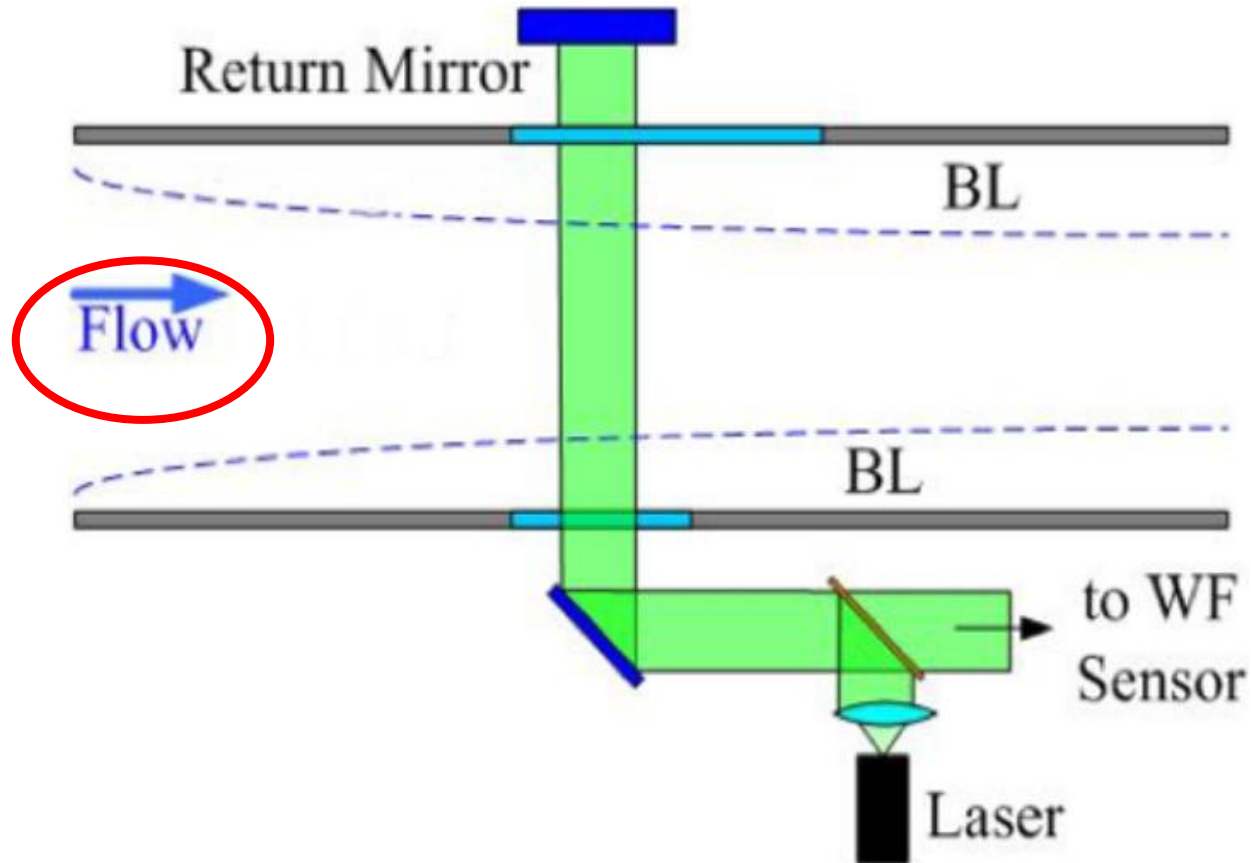
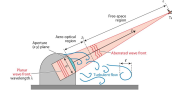
$(\hat{L}_0, \hat{r}_0, \hat{v}_x, \hat{v}_y, \hat{\alpha})$

Generate



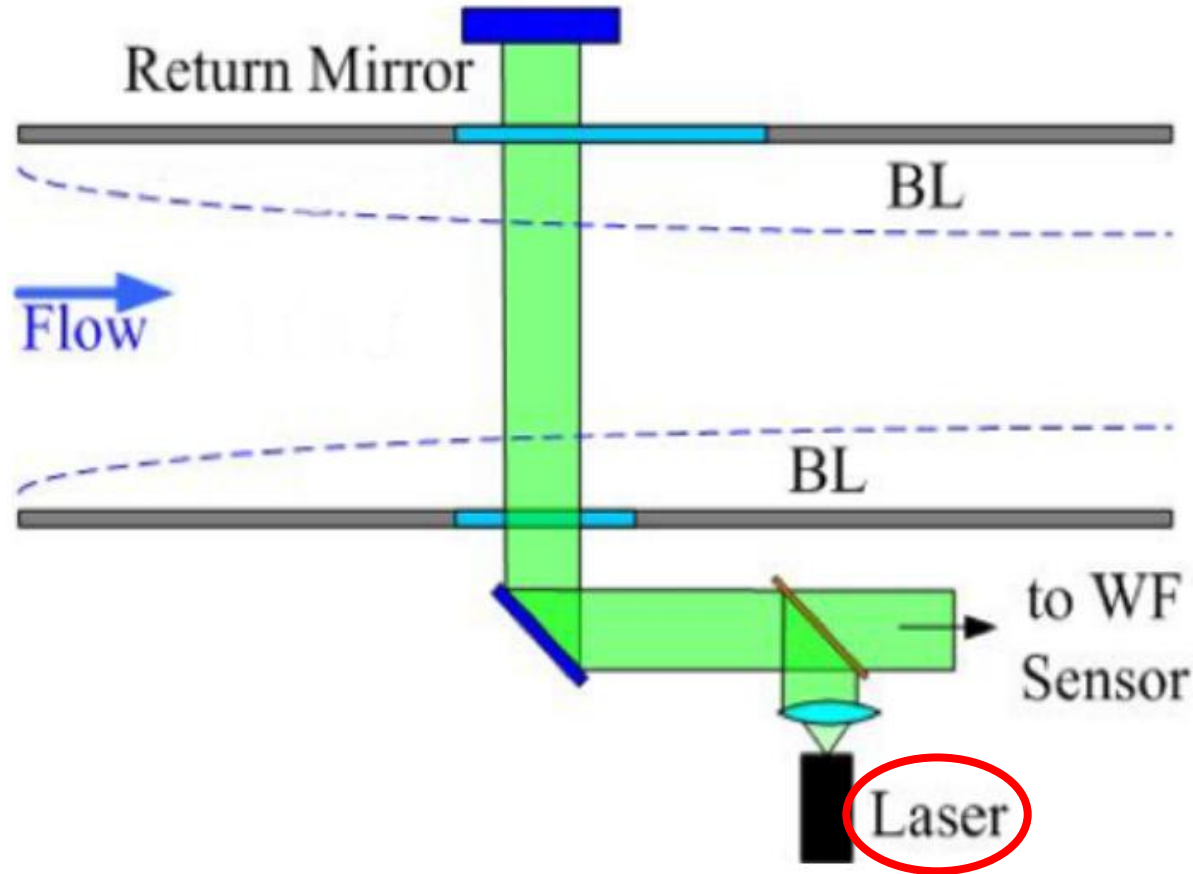
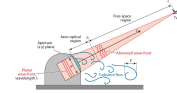
Boiling Flow

Measured Data: Wind Tunnel Experiment



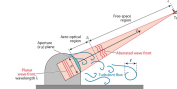
[4] M. R. Kemnetz and S. Gordeyev, "Optical investigation of large-scale boundary-layer structures", *54th AIAA Aerospace Sciences Meeting*, 4 - 8 Jan 2016, San Diego, California, AIAA Paper 2016-1460.

Measured Data: Wind Tunnel Experiment

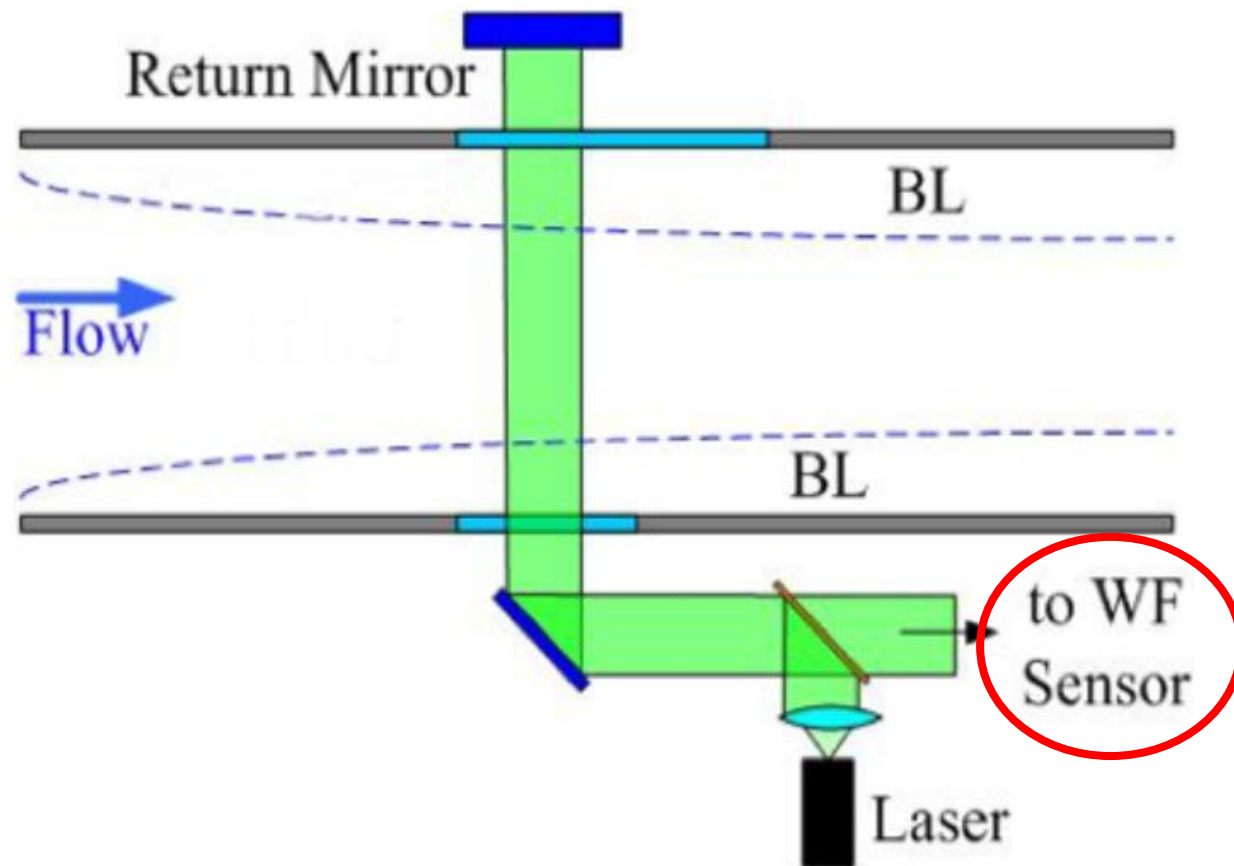


[4] M. R. Kemnetz and S. Gordeyev, "Optical investigation of large-scale boundary-layer structures", *54th AIAA Aerospace Sciences Meeting*, 4 - 8 Jan 2016, San Diego, California, AIAA Paper 2016-1460.

Measured Data: Wind Tunnel Experiment

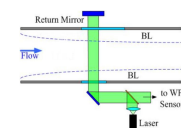


The result of the experiment is a **time-series** of *aero-optic phase screens*.



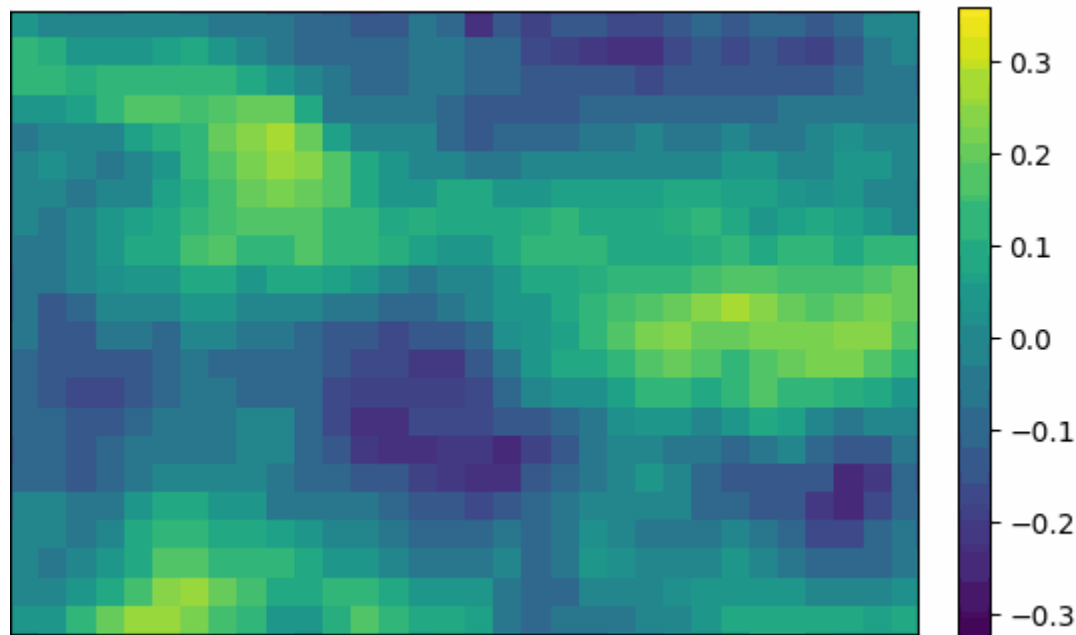
[4] M. R. Kemnetz and S. Gordeyev, "Optical investigation of large-scale boundary-layer structures", 54th AIAA Aerospace Sciences Meeting, 4 - 8 Jan 2016, San Diego, California, AIAA Paper 2016-1460.

Experimental Data: Visualization

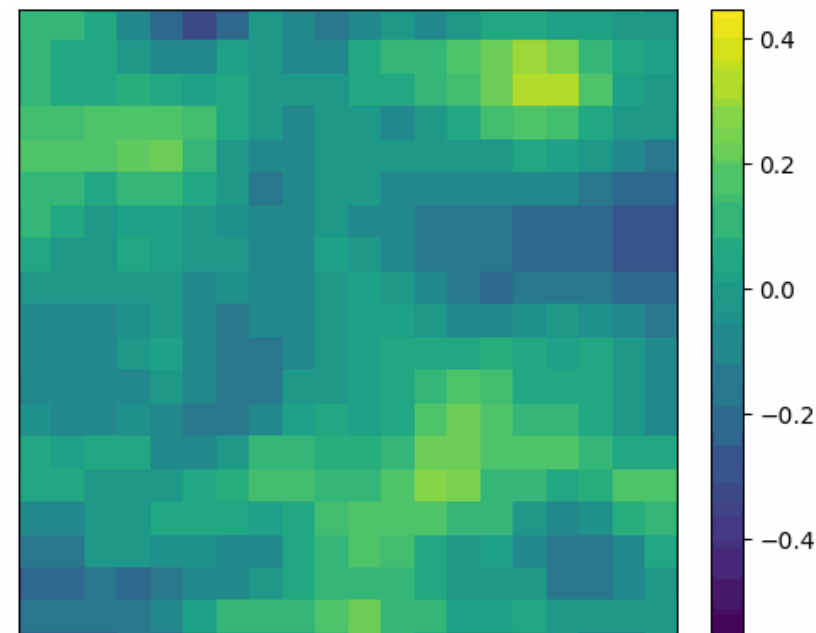


The boundary layer data is *highly convective*.

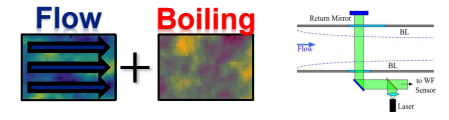
Data Set F06



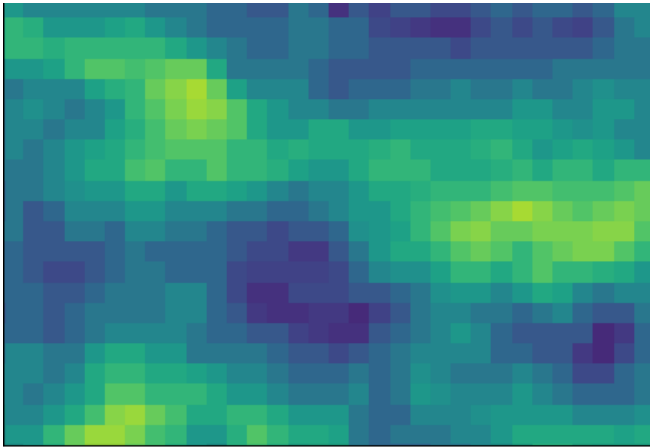
Data Set F12



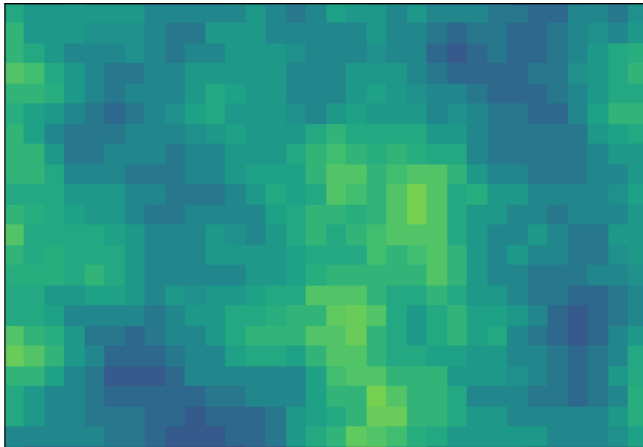
Parameter Estimation from Measured Data



Measured Phase Screens



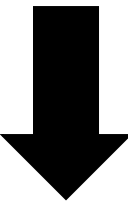
Synthetic Phase Screens



Compare

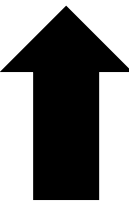


Estimate



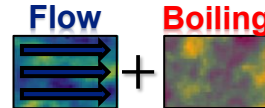
$$(\hat{L}_0, \hat{r}_0, \hat{v}_x, \hat{v}_y, \hat{\alpha})$$

Generate



Boiling Flow

Boiling Flow Parameters

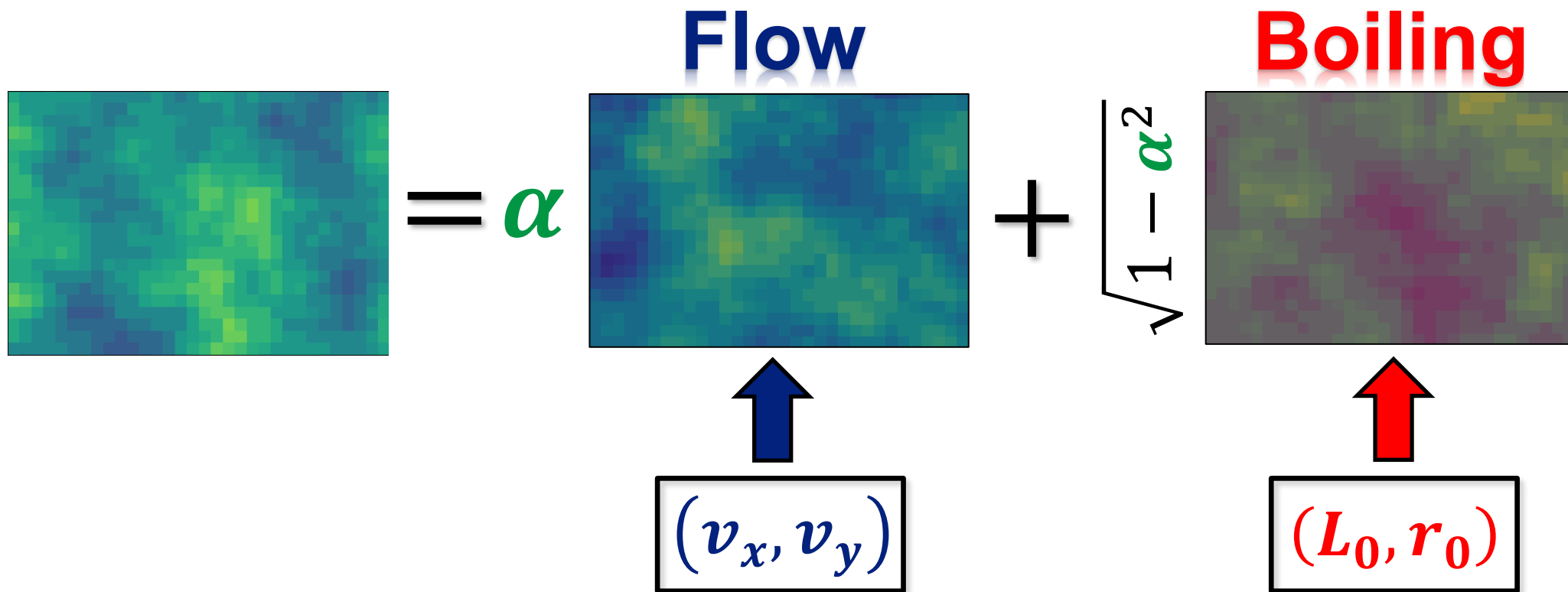


➤ L_0 : outer scale

➤ r_0 : Fried coherence length

➤ (v_x, v_y) : flow velocity components

➤ α : *boiling coefficient*

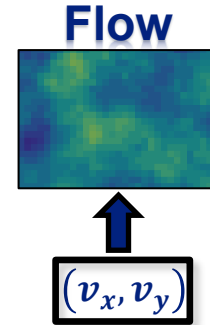


Parameter Estimation Algorithm: Outline

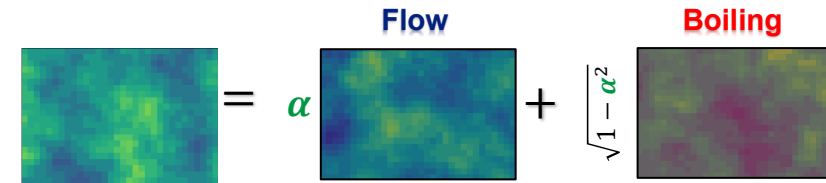
1. Boiling parameters (L_0, r_0)



2. Flow parameters (v_x, v_y)



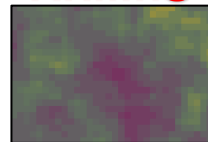
3. Boiling coefficient α



Parameter Estimation Algorithm: Outline

1. Boiling parameters (L_0, r_0)

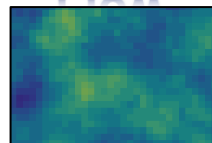
Boiling



(L_0, r_0)

2. Flow parameters (v_x, v_y)

Flow

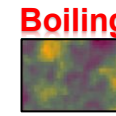


(v_x, v_y)

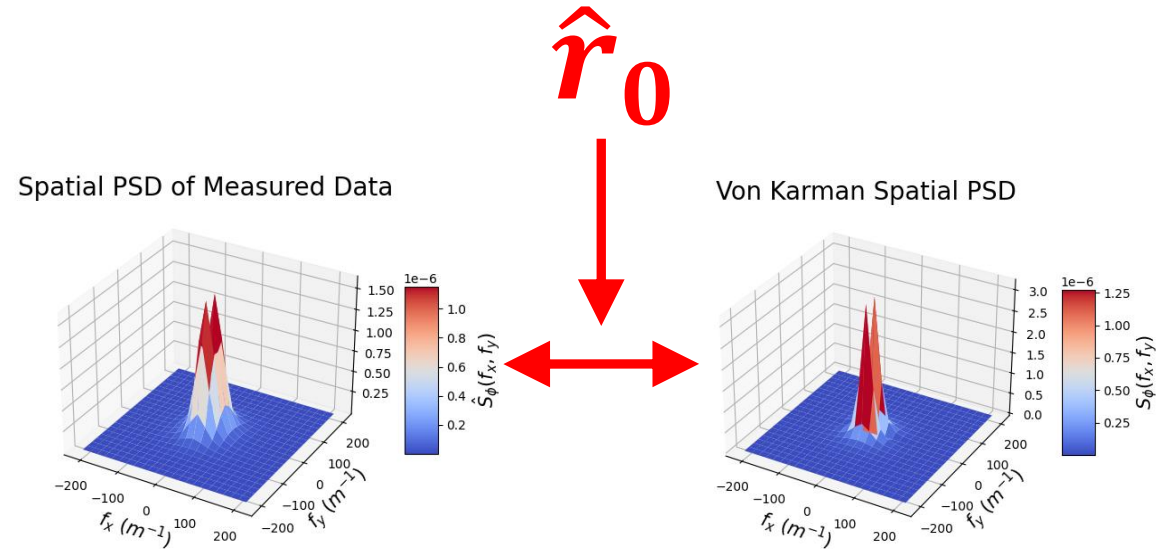
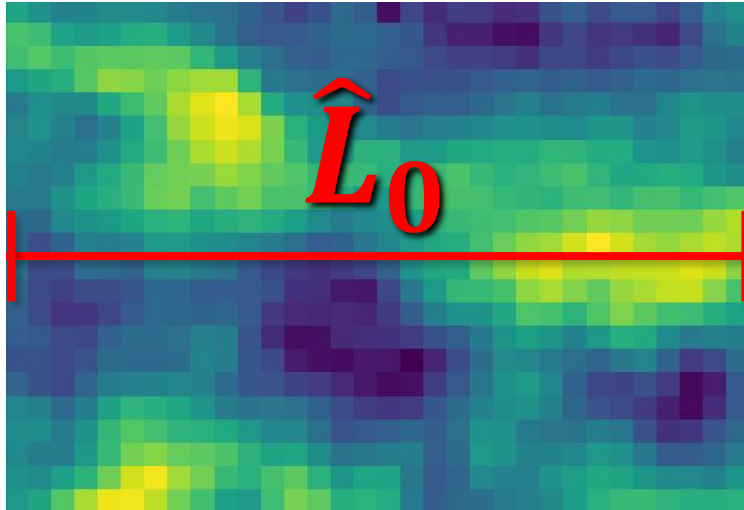
3. Boiling coefficient α

$$\text{Result Map} = \alpha \text{ Flow} + \sqrt{1 - \alpha^2} \text{ Boiling}$$
The diagram shows the final result map as a linear combination of the flow and boiling parameter maps. On the left is the result map (green/yellow/blue). This is followed by an equals sign, then the Greek letter alpha, then the flow map (blue/green/yellow), followed by a plus sign, then the square root of (1 minus alpha squared), and finally the boiling map (purple/blue/green). The flow and boiling maps are labeled 'Flow' and 'Boiling' respectively.

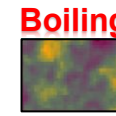
Estimating the Boiling Parameters (L_0, r_0)



- Set $\hat{L}_0$ to the length of the aperture (m).
- Estimate $\hat{r}_0$ to fit the *von Karman PSD* to the measured data's spatial PSD, $\hat{S}_\phi(f_x, f_y)$.

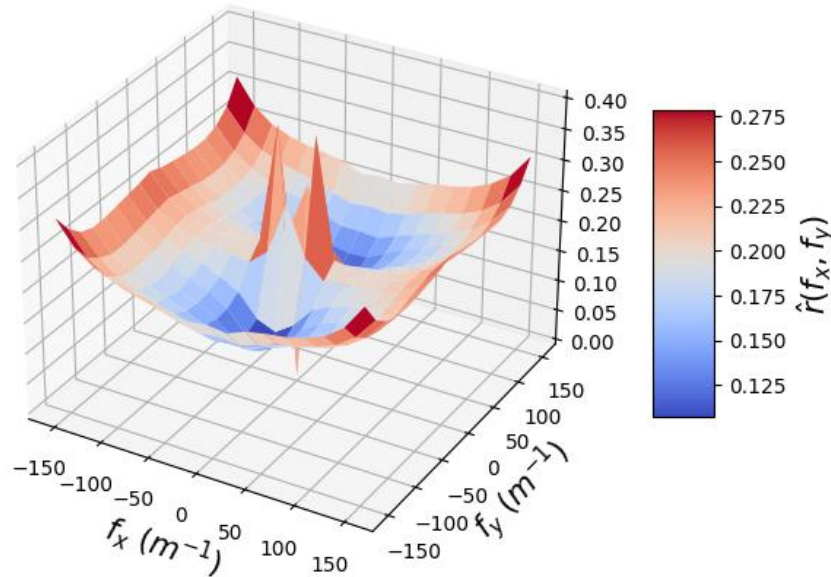


Estimating the Boiling Parameters (L_0, r_0)

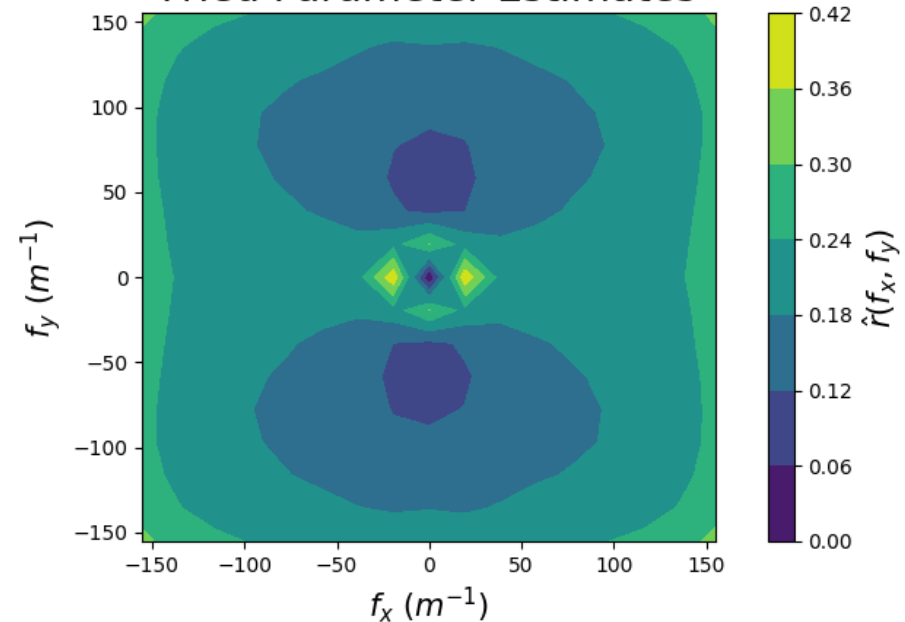


- This gives us an estimate $\hat{r}(f_x, f_y)$.
- We take $\hat{r}_0$ as the average of $\hat{r}(f_x, f_y)$ over (f_x, f_y) .

Fried Parameter Estimates



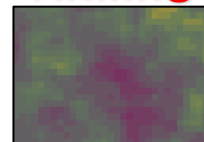
Fried Parameter Estimates



Parameter Estimation Algorithm: Outline

1. Boiling parameters (L_0, r_0)

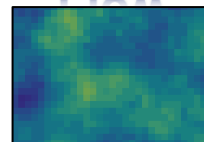
Boiling



(L_0, r_0)

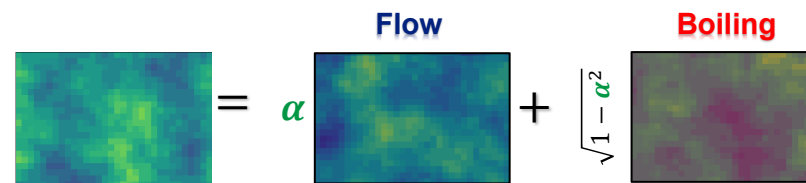
2. Flow parameters (v_x, v_y)

Flow

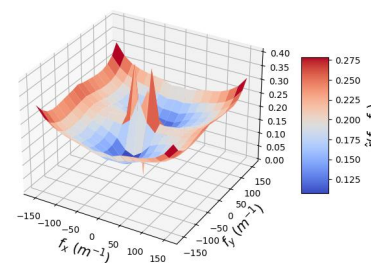


(v_x, v_y)

3. Boiling coefficient α

$$\text{Total} = \alpha \cdot \text{Flow} + \sqrt{1 - \alpha^2} \cdot \text{Boiling}$$
The diagram illustrates the combination of Flow and Boiling parameters. It shows a 'Flow' heatmap (blue to yellow) and a 'Boiling' heatmap (purple to yellow) being combined via a weighted sum. The result is a 'Total' heatmap (green to yellow). The equation is: Total = alpha * Flow + sqrt(1 - alpha^2) * Boiling.

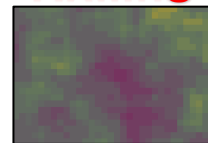
Fried Parameter Estimates



Parameter Estimation Algorithm: Outline

1. Boiling parameters (L_0, r_0)

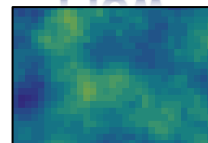
Boiling



(L_0, r_0)

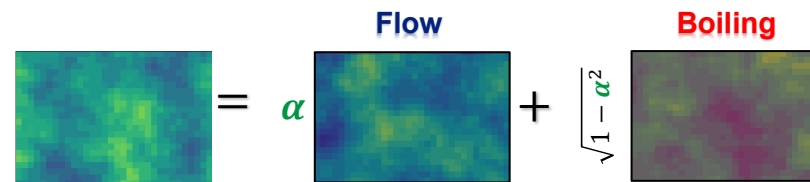
2. Flow parameters (v_x, v_y)

Flow

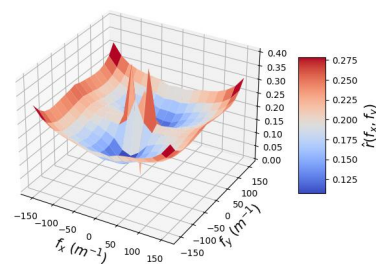


(v_x, v_y)

3. Boiling coefficient α

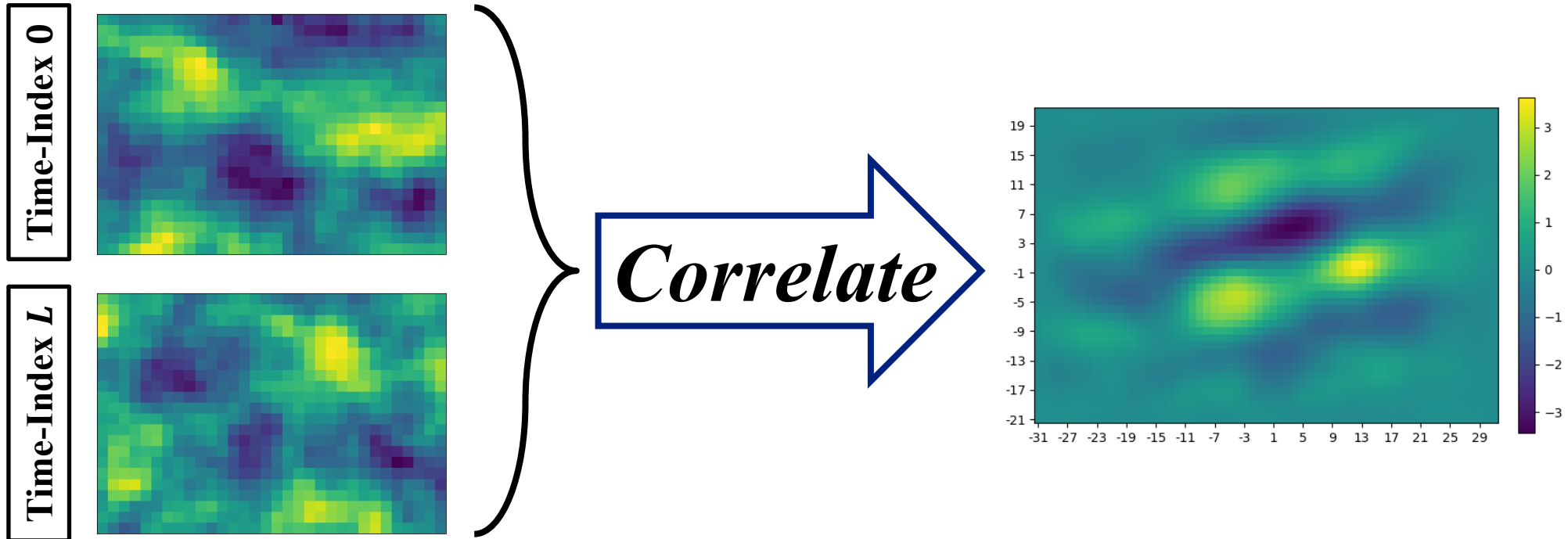
$$\text{Total} = \alpha \cdot \text{Flow} + \sqrt{1 - \alpha^2} \cdot \text{Boiling}$$
The diagram illustrates the combination of Flow and Boiling parameters to form the total parameter estimate. It shows a large heatmap on the left, followed by an equals sign, then the word 'Flow' above a heatmap, a plus sign, the expression 'sqrt(1 - alpha^2)' above another plus sign, and finally the word 'Boiling' above a heatmap. The 'Flow' and 'Boiling' heatmaps are smaller than the 'Total' heatmap.

Fried Parameter Estimates

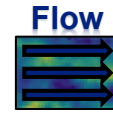


Estimating the Flow Parameters (v_x, v_y)

Search for the *pixel shifts* (i, j) which maximize the *cross-correlation* $R(i, j; L)$ (with time-lag L).

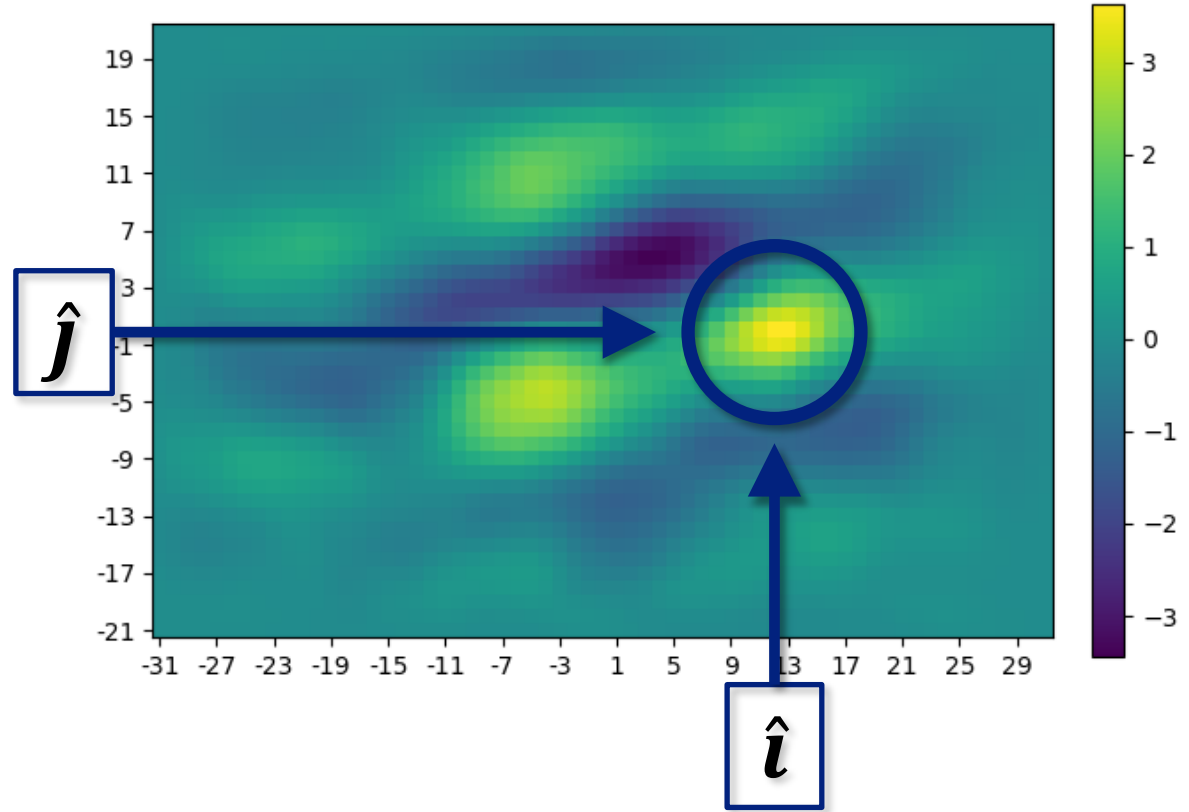


Estimating the Flow Parameters (v_x, v_y)



- The maximal *pixel shift* $(\hat{i}, \hat{j})$ is where the *correlation is the largest*.
- The *flow velocity* estimates divide by the time-lag L :

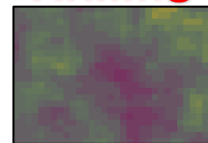
$$(\hat{v}_x, \hat{v}_y) = \frac{1}{L} (\hat{i}, \hat{j})$$



Parameter Estimation Algorithm: Outline

1. Boiling parameters (L_0, r_0)

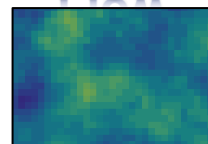
Boiling



(L_0, r_0)

2. Flow parameters (v_x, v_y)

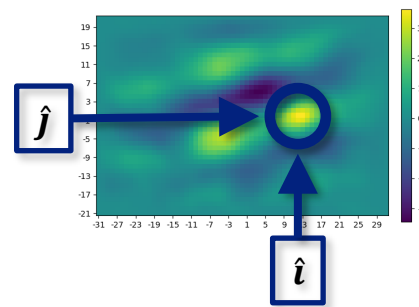
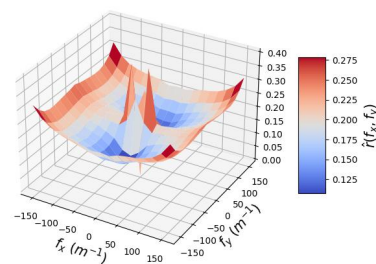
Flow



(v_x, v_y)

3. Boiling coefficient α

Fried Parameter Estimates

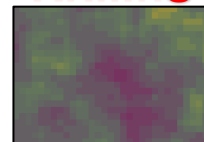


$$\text{Total Map} = \alpha \cdot \text{Flow Map} + \sqrt{1 - \alpha^2} \cdot \text{Boiling Map}$$

Parameter Estimation Algorithm: Outline

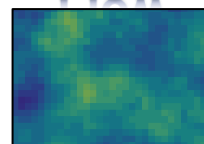
1. Boiling parameters (L_0, r_0)

Boiling



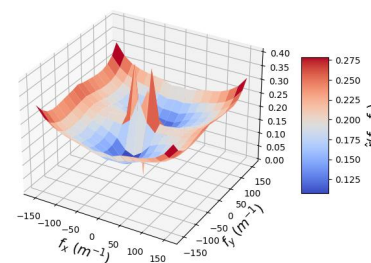
(L_0, r_0)

Flow

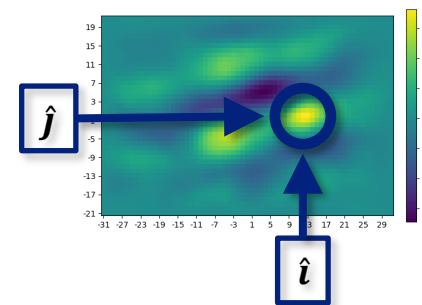


(v_x, v_y)

Fried Parameter Estimates



2. Flow parameters (v_x, v_y)



3. Boiling coefficient α

$$\text{Flow} = \alpha \text{ Flow} + \sqrt{1 - \alpha^2} \text{ Boiling}$$
The diagram illustrates the equation for the boiling coefficient α . It shows a 2D heatmap on the left, followed by an equals sign, then the word "Flow" above a 2D heatmap, a plus sign, and the expression $\sqrt{1 - \alpha^2}$ above another 2D heatmap labeled "Boiling".

Estimate the Boiling Coefficient α +

➤ We use a **least-squares fitting** to compute $\hat{\alpha}$, given the flow velocity components $(\hat{v}_x, \hat{v}_y)$:

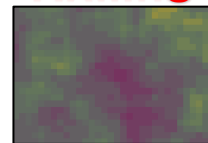
$$\hat{\alpha} = \underset{\alpha}{\operatorname{argmin}} \left\{ \sum_n \|\phi_n - \alpha \textit{Flow}(\phi_{n-1})\|^2 \right\}$$

➤ $(\hat{v}_x, \hat{v}_y, \hat{\alpha})$ fit the temporal statistics of the measured data.

Parameter Estimation Algorithm: Outline

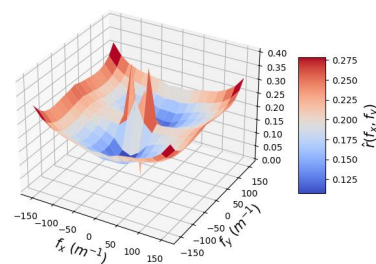
1. Boiling parameters (L_0, r_0)

Boiling



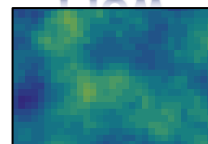
(L_0, r_0)

Fried Parameter Estimates

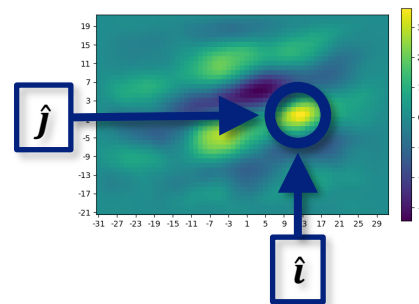


2. Flow parameters (v_x, v_y)

Flow



(v_x, v_y)

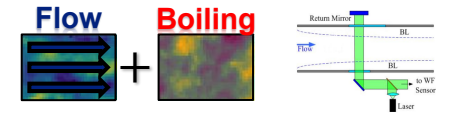


3. Boiling coefficient α

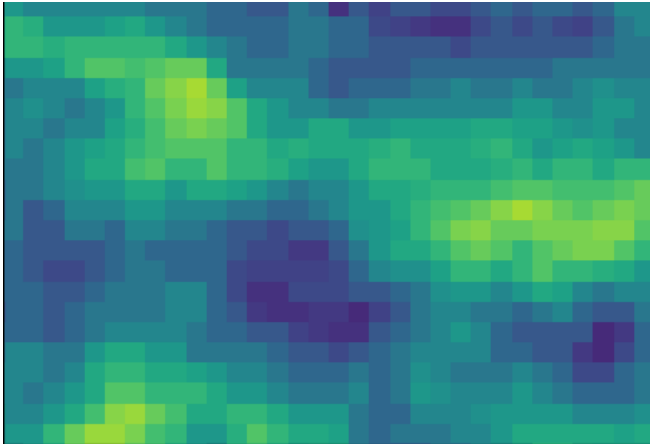
$$\text{Flow} = \alpha \text{Flow} + \sqrt{1 - \alpha^2} \text{Boiling}$$

$$\hat{\alpha} = \underset{\alpha}{\operatorname{argmin}} \left\{ \sum_n \|\phi_n - \alpha \text{Flow}(\phi_{n-1})\|^2 \right\}$$

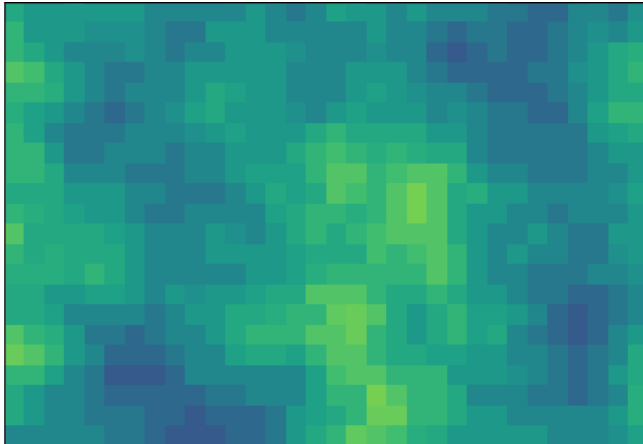
Parameter Estimation from Measured Data



Measured Phase Screens



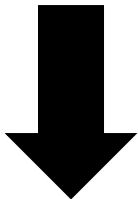
Synthetic Phase Screens



Compare

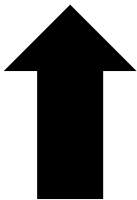
A black double-headed horizontal arrow is centered within a green rectangular border.

Estimate



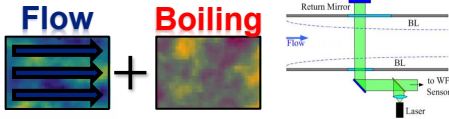
$$\left(\hat{L}_0, \hat{r}_0, \hat{v}_x, \hat{v}_y, \hat{\alpha} \right)$$

Generate

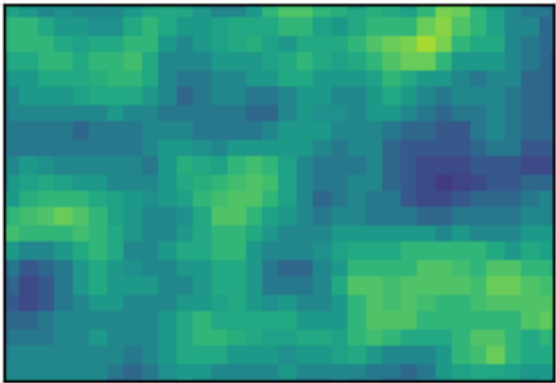


Boiling Flow

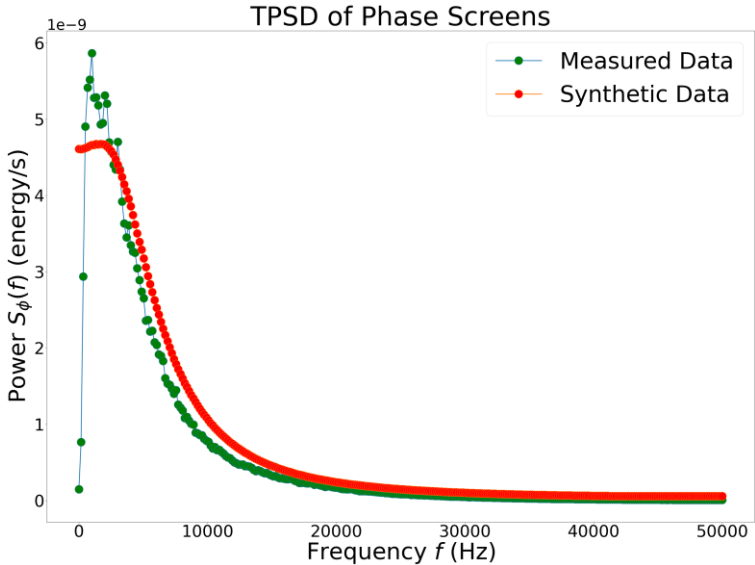
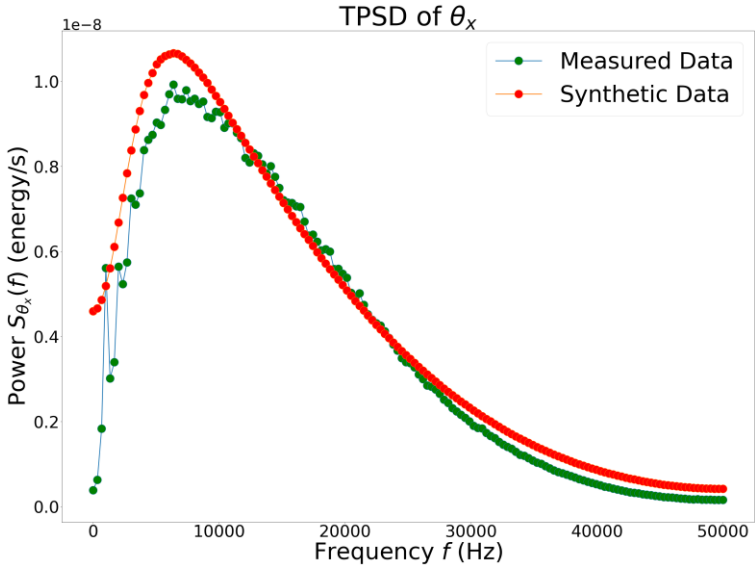
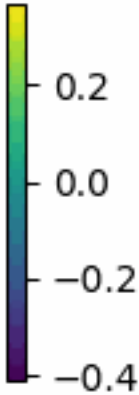
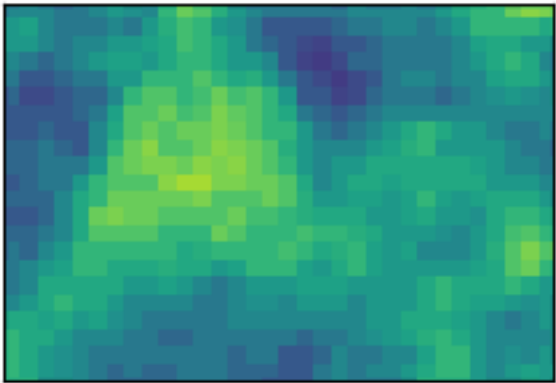
Results: Data Set F06



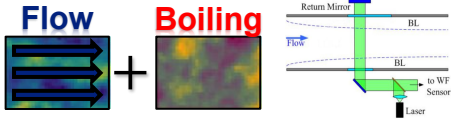
Measured



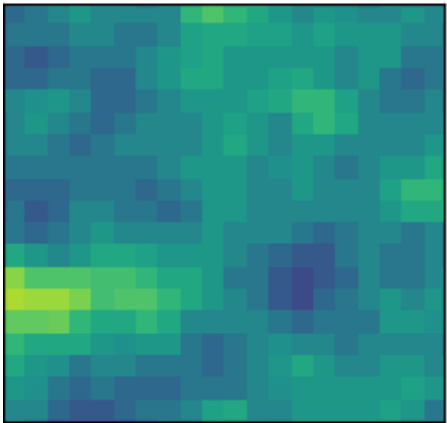
Synthetic



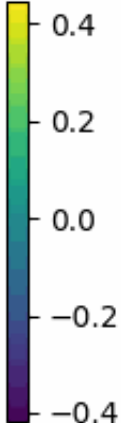
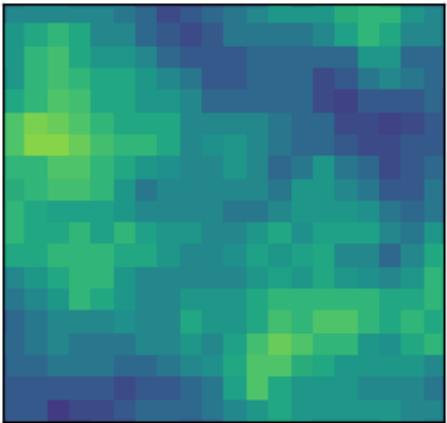
Results: Data Set F12



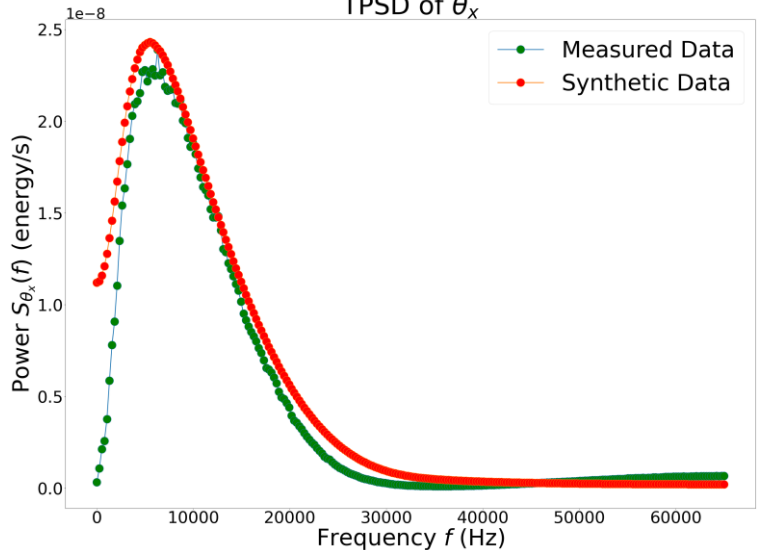
Measured



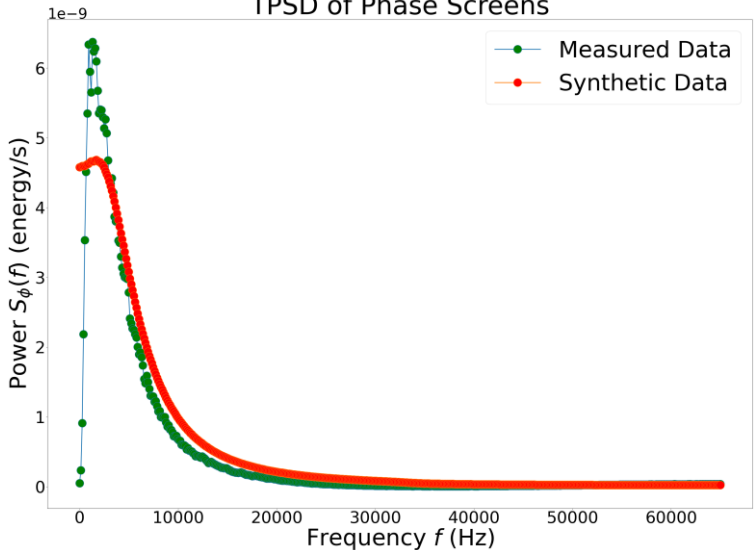
Synthetic



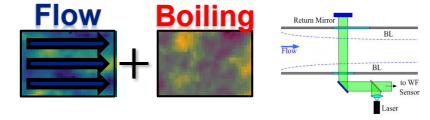
TPSD of θ_x



TPSD of Phase Screens



2-D Kolmogorov Spatial Structure Function

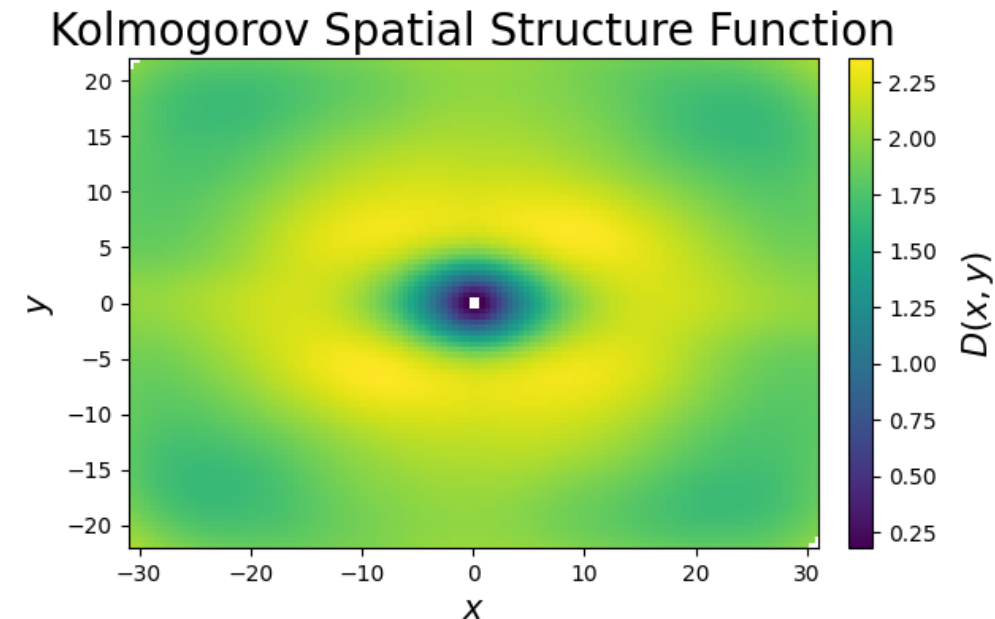


- The **2-D Kolmogorov spatial structure function**

$$D_{\phi}(x, y) = \mathbb{E} \left[\left(\phi(x_0 + x, y_0 + y) - \phi(x_0, y_0) \right)^2 \right]$$

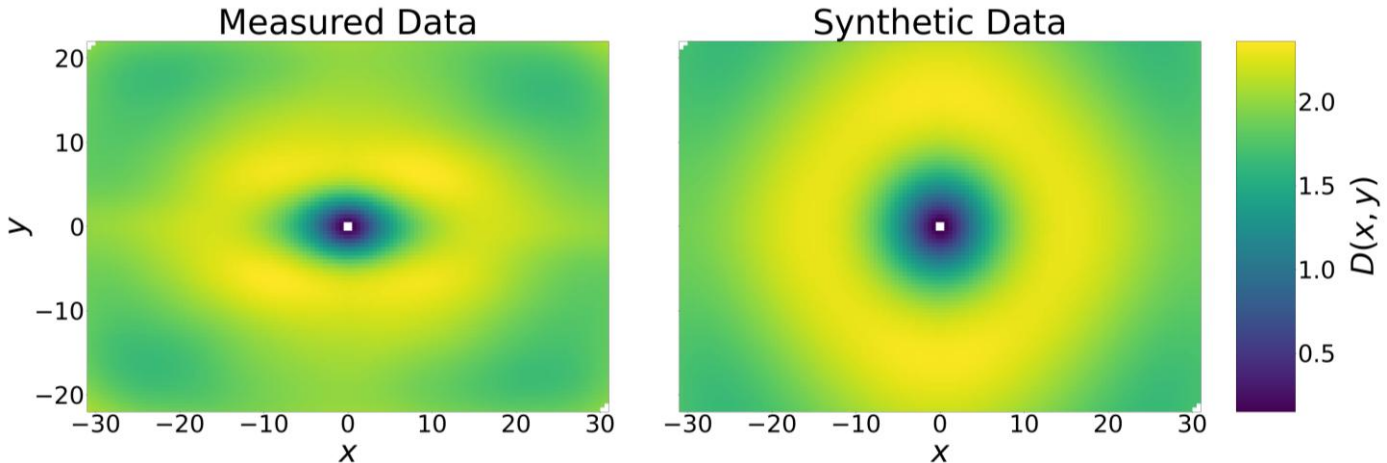
evaluates the *spatial statistics* of the phase screens.

- This function depends on the **(x, y) difference**, instead of just the **separation distance r** .

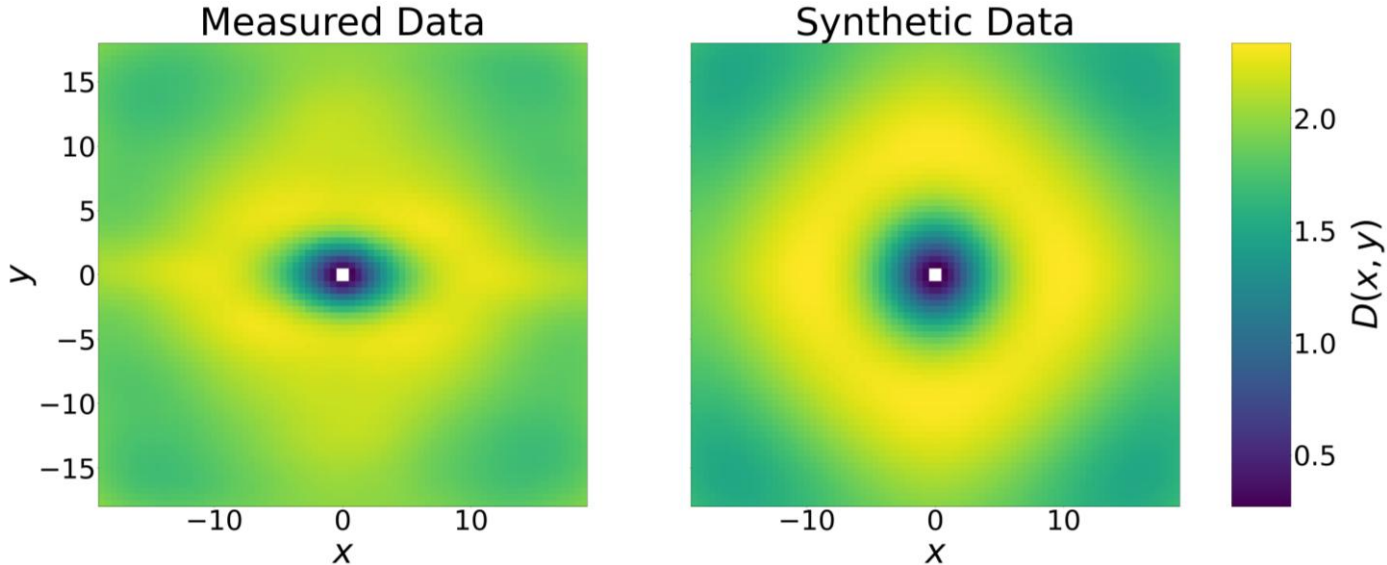


Spatial Statistics of Measured and Synthetic Data

*Data Set
F06*

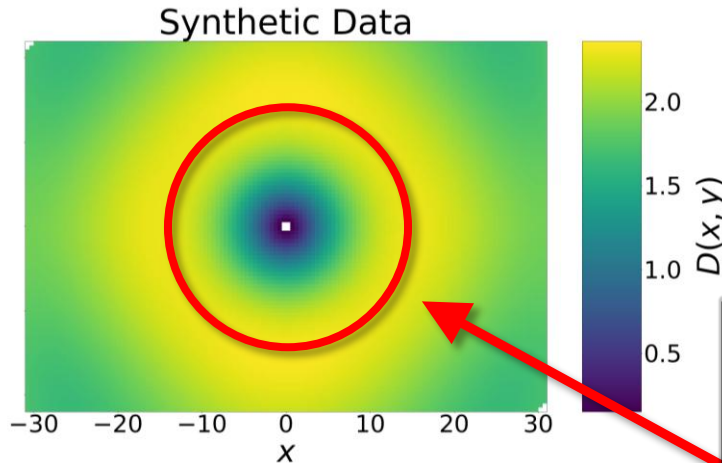
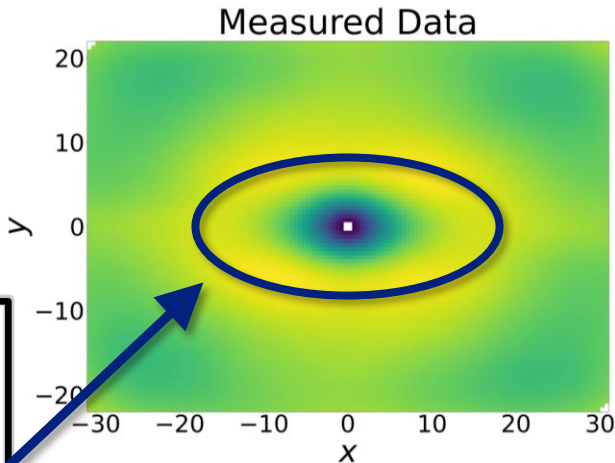


*Data Set
F12*

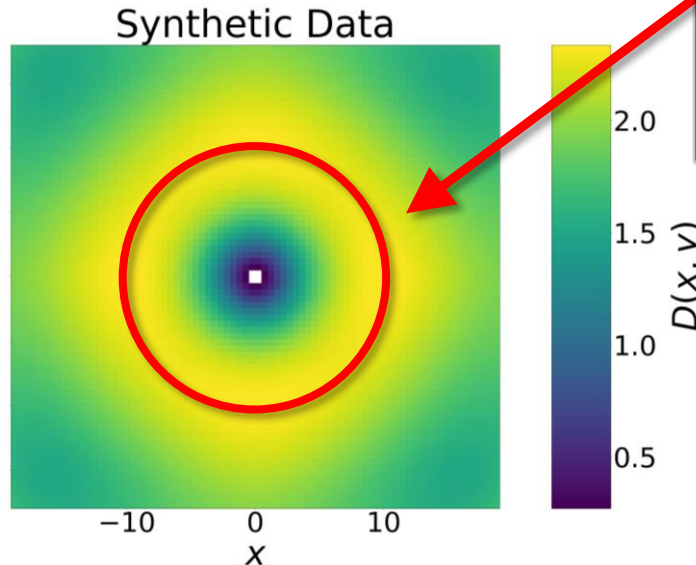
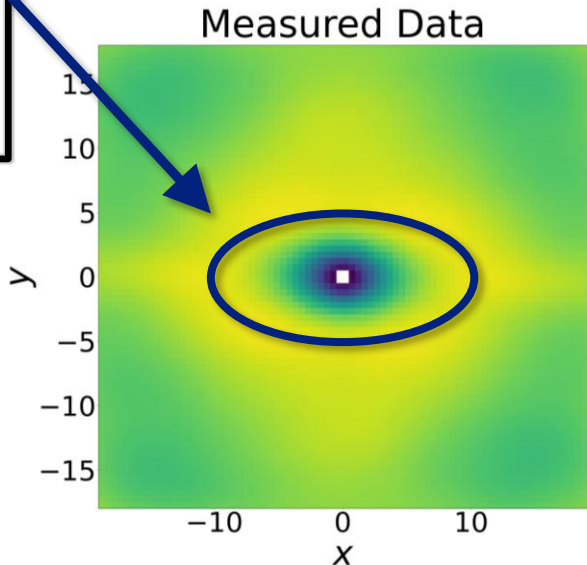


Spatial Statistics of Measured and Synthetic Data

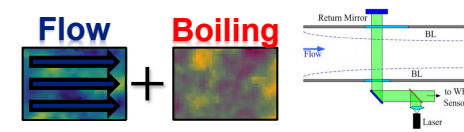
Contours
are
ellipses



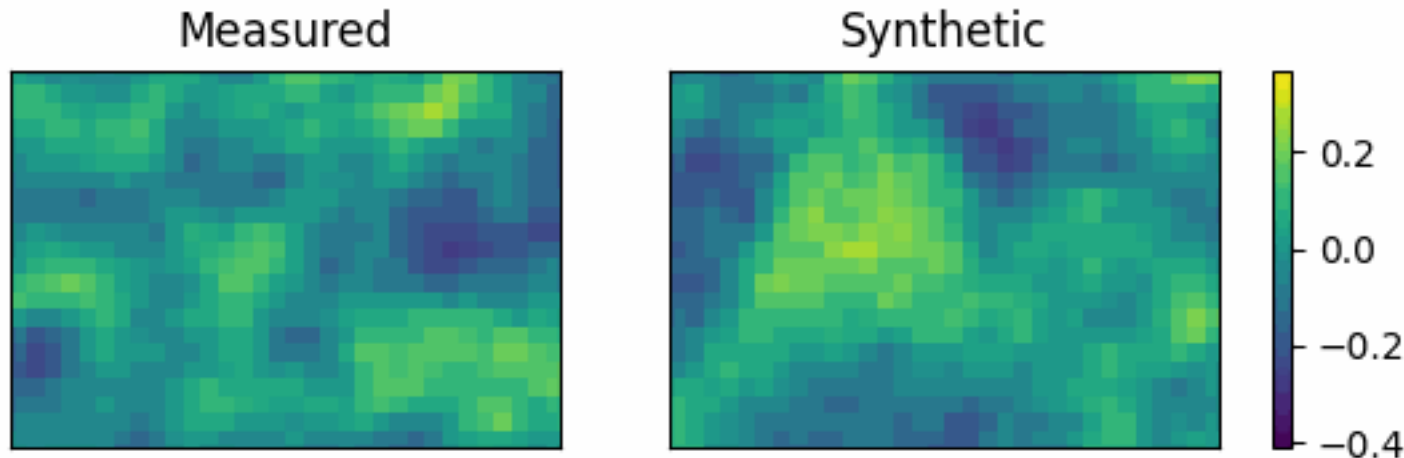
Contours
are
circles



Conclusion



- We *estimated* the **Boiling Flow** parameters from measured *aero-optic* phase screens.
- The resulting synthetic phase screens **reasonably match the temporal statistics** of the *slopes* of the measured phase screens, but **do not match the spatial statistics** of the measured phase screens.



References

- [1] E. J. Jumper, S. Gordeyev, and M. R. Whiteley, “Aero-optical effects,” in *Aero-Optical Effects*, (John Wiley Sons, Incorporated, United States, 2023)

- [2] M. Wang, A. Mani, and S. Gordeyev, “Physics and Computation of Aero-Optics,” *Annual Review of Fluid Mechanics*, Vol. 44, No. 1, 2012, pp. 299–321.

- [3] Srinath, S., Poyneer, L. A., Rudy, A. R., and Ammons, S. M., “Computationally efficient autoregressive method for generating phase screens with frozen flow and turbulence in optical simulations,” *Opt. Express* 23, 33335–33349 (Dec 2015)

- [4] M. R. Kemnetz and S. Gordeyev, "Optical investigation of large-scale boundary-layer structures", *54th AIAA Aerospace Sciences Meeting*, 4 - 8 Jan 2016, San Diego, California, AIAA Paper 2016-1460.