A STUDY OF THE REPRESENTATIONS SUPPORTED BY THE ORBIT CLOSURE OF THE DETERMINANT

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1. Abstract

Let $v$ be a complex vector space of dimension $m$ and let $E := v \otimes v^* = \text{End} v$. Consider $\det \in Q := S^m(E^*)$, where $\det$ is the function taking determinant of any $X \in \text{End} v$. The group $G = \text{GL}(E)$ canonically acts on $Q$. Let $\mathcal{X}$ be the $G$-orbit closure of $\det$ inside $Q$. Then, $\mathcal{X}$ is a closed (affine) subvariety of $Q$ which is stable under the standard homothetic action of $\mathbb{C}^*$ on $Q$. Thus, its affine coordinate ring $\mathbb{C}[\mathcal{X}]$ is nonnegatively graded $G$-algebra over the complex numbers $\mathbb{C}$.

The aim of this talk is to study $\mathbb{C}[\mathcal{X}]$ as a $G$-module. The work is motivated by the geometric approach initiated by Mulmuley-Sohoni to solve the Valiant’s conjecture in Geometric Complexity Theory. We relate its study with the Latin Square Conjecture due to Alon-Tarsi and an equivalent formulation due to Huang-Rota called the column Latin square conjecture.