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| Name        |             |
| Instructor: | Uli Walther |

Math 416 Fall 2016  
Practice Exam Questions

You are not allowed to use books or notes. Calculators are permitted. Full credit is given for complete correct solutions. *Please show all your work. No credit is given for unsupported answers*

1. (For all parts, write down the expression without evaluating it.) A bridge hand (13 cards) is drawn at random from a standard deck of 52 cards.

(a) How many different hands are there?

(b) What is the probability that the hand contains 8 red cards (hearts or diamonds) and 5 black cards (clubs or spades)?

(c) What is the probability that the hand contains an ace or a king?

2. An urn contains 5 blue balls, 3 green balls, and an unknown number of red balls. 2 balls are drawn from the urn (without replacement). If the probability that the draw consists of 2 blue balls is  $\frac{2}{9}$ , how many red balls are in the urn?

3. Among the 100 patrons of a bar, 42 are fans of basketball, 41 are fans of baseball, 65 are fans of either soccer or basketball, 60 are of either soccer or baseball, and 75 are of either basketball or baseball. If only 1 patron is a fan of all three sports, and if 10 fans don't care about any of the three sports, how many patrons are soccer fans?

4. A student is taking midterms in Linear Algebra, Probability, and Analysis. The probabilities of getting an A in Linear Algebra, Probability, and Analysis are 0.3, 0.5, and 0.25, respectively. Given that grades on the midterms in these courses are mutually independent, the probability that a student will get at least one A.

5. The probability density function  $f_X$  for a random variable  $X$  is given by

$$f_X(x) = \begin{cases} 1 - |x|, & |x| \leq 1, \\ 0 & \text{otherwise.} \end{cases}$$

Find the distribution function  $F_X(x)$  and the variance  $Var(X)$ .

6. Three identically looking coins lie on the table. Coin I is fair, coin II comes up tails 70% of the time, and coin III comes up tails 40% of the time. A coin is picked at random and flipped once. The outcome of the flip was heads. What is the probability that coin I was chosen?

7. 50 distinct numbers are chosen at random from  $\{1, \dots, 100\}$ . What is the probability that the smallest number among the chosen is  $k$ ? What is the range of value for  $k$ ?

8. Suppose that a probability that a game of baseball lasts  $N$  innings is

$$P(N = n) = \begin{cases} a \times \left(\frac{1}{3}\right)^{n-9}, & : n \geq 9 \\ 0 & : \text{otherwise.} \end{cases}$$

(a) Find  $a$ .

(b) What is the probability that a game ends in less than 12 innings if it went into extra-innings (i.e.,  $N > 9$ )?

9. Assume that the life of a computer processor is an exponential random variable, and that the processor lasts on average 4 years.
- What is the probability that a processor will die within the first 3 years?
  - Assuming that a processor lasted for 3 years, what is the probability that it will work for another 2 years?

10. Suppose  $X$  and  $Y$  are independent exponential random variables with parameter  $\lambda = 1$ . Find the distribution function for  $Z = \frac{X}{Y}$ .

11. A customer is expecting one package to be delivered by UPS and another package to be delivered by Fedex. The UPS package is expected to arrive anytime between 1 PM and 6 PM. The Fedex package is expected to be delivered anytime between 3 PM and 5 PM. Find the probability that the UPS package will arrive before the package from Fedex.

12. The joint probability density function of  $X$  and  $Y$  is given by

$$f_{XY}(x, y) = c(y^2 - x^2), -y \leq x \leq y, 0 < y < 3.$$

- a. Find  $c$ .
- b. Find the marginal densities  $f_X$  and  $f_Y$
- c. Are  $X$  and  $Y$  independent?

13. 1000 cards are drawn with replacement from the standard 52 deck of cards. Then ace of spades is removed from the deck, and another 1000 cards are drawn with replacement from the short deck. Approximate the probability that at least 140 of these 2000 drawn cards are aces.

14. Let  $Y$  be an exponential random variable with  $\lambda = 1$ . Find the density function for  $W = Y^3$ .