Math 460: Homework # 10.

This assignment covers up to Theorem 40 in the course notes (so there is no new reading for this assignment).

1. (Use Geometer's Sketchpad.) In Figure 1, find an equation relating $\angle 1$, arc ABC and arc DEF.

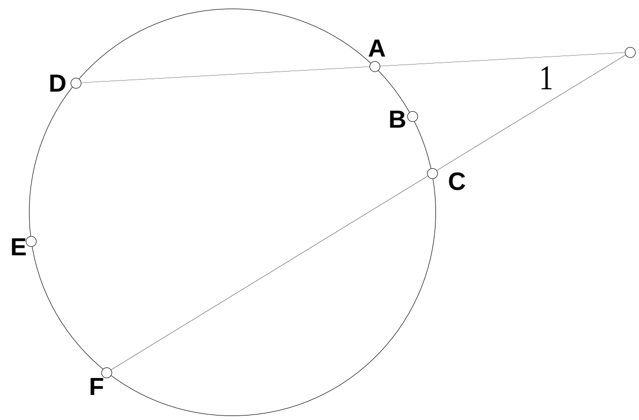

Figure 1

2. (Use Geometer's Sketchpad.) Begin with a point A and four lines ℓ, m, n and p that go through A. Next, hide the points other than A used to construct these lines (this is important). Choose a point B on ℓ and a point C on m, and let D and E be the intersections of $\overleftrightarrow{BC}$ with n and p respectively. Find a combination of the distances BC, CD, BE and DE which doesn't change when the points B and C are moved (leaving A and the lines ℓ, m, n and p fixed). Hint: the combination is the product of two of the lengths divided by the product of the two others.

3. (In this problem we prove the fact that you discovered in Problem 1 of Assignment 9). In Figure 2, prove that $\angle 1 = \frac{1}{2}\text{arc}ABC + \frac{1}{2}\text{arc}DEF$. (Hint: draw in one extra line.)

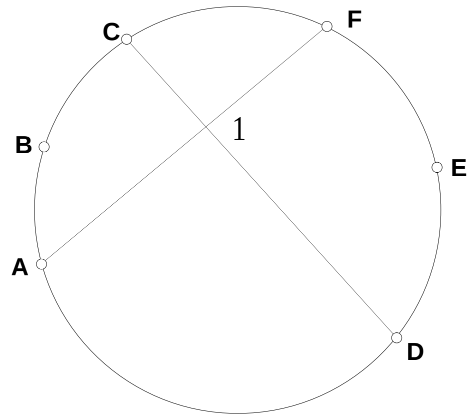

Figure 2

4. (In this problem we prove the fact that you discovered in Problem 2 of Assignment 9). See Figure 3. Given: $\angle 1 = \angle 2$, $\angle 3 = \angle 4$, $\angle 5 = \angle 6$, $\angle 7 = \angle 8$. To prove: $AB + CD = BC + AD$.

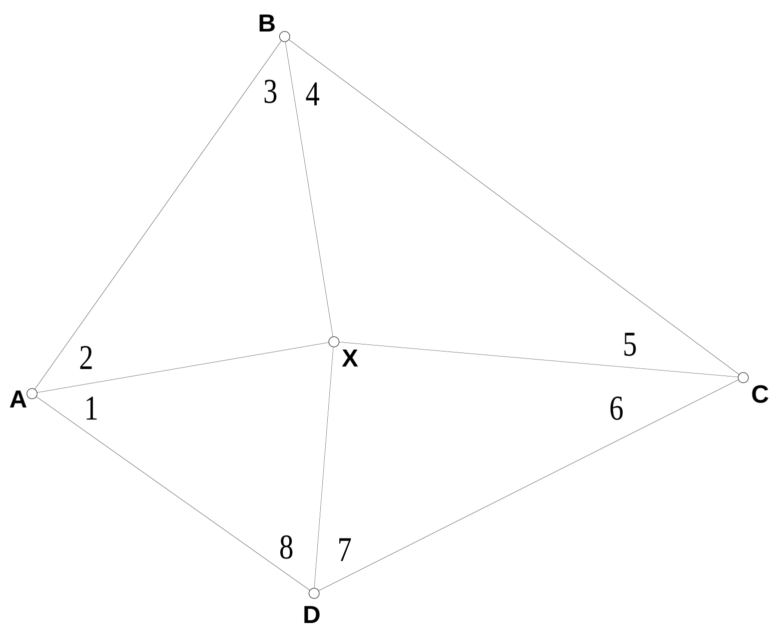

Figure 3

5. Prove the case of Theorem 40 which is illustrated in Figure 42.

6. Euclid's proof of Proposition 11 uses Proposition 8 as one ingredient. Find a proof of Proposition 11 which uses only facts from Euclid that come *before* Proposition 6 (and nothing from the course notes.)

7. (In this problem we complete the proof of the fact that you discovered in Problem 1 of Assignment 7.) Prove that in Figure 4 (or any other analogous picture in which the points P, Q, R, P', Q' and R' are chosen differently) the points X, Y and Z are collinear. (Hint: redo Problem 4 of Assignment 9, but this time use Theorem 32 to keep track of the signs—this may be easy, depending on how you did Problem 4 of Assignment 9 the first time. Then use Theorem 33.)

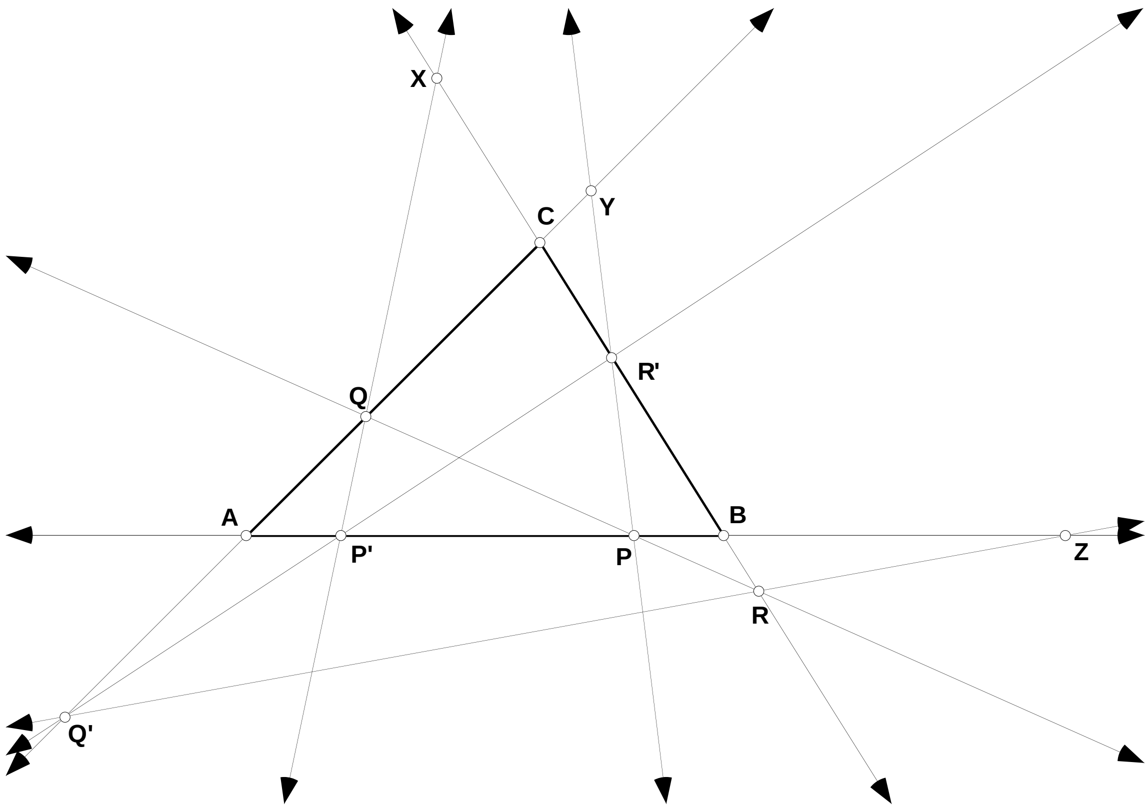

Figure 4

8. (In this problem we continue to prove what you discovered in Problem 2 of Assignment 6.) See Figure 5. Given: G is the centroid of $\triangle ABC$ and U, V, W, X, Y and Z are the centroids of the six "little triangles." To prove: the area of $\triangle DZU$ is 1/16 of the area of $\triangle DEF$. (You may use anything that you have already proved about this picture in earlier homework. But be specific in saying what you are using and when it was shown.)

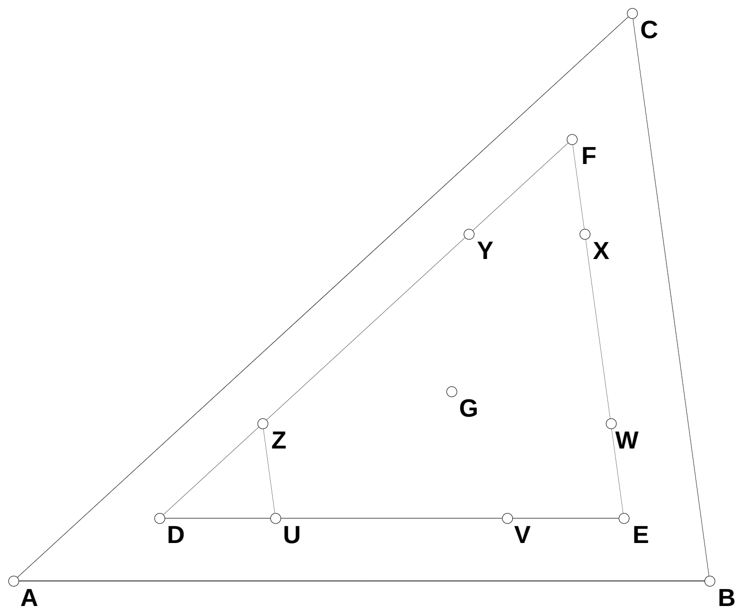

Figure 5

9. (See Figure 6.) Given: $\angle ABC$ is a right angle, and $ABDE$ and $ACFG$ are squares.
To prove: $BG = CE$ and $BG \perp CE$.

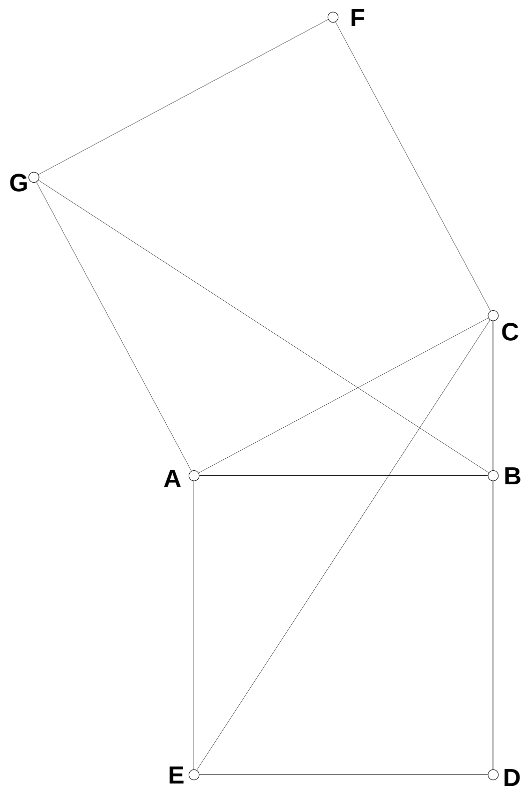

Figure 6

10. (See Figure 7.) This is a problem about three-dimensional geometry. Given: A, B and C are in the indicated plane, RBS is a straight line, $RB = SB$, $AB \perp RS$, and $\angle CAR = \angle CAS$. To prove: $\angle ACR = \angle ACS$ and $BC \perp RS$.

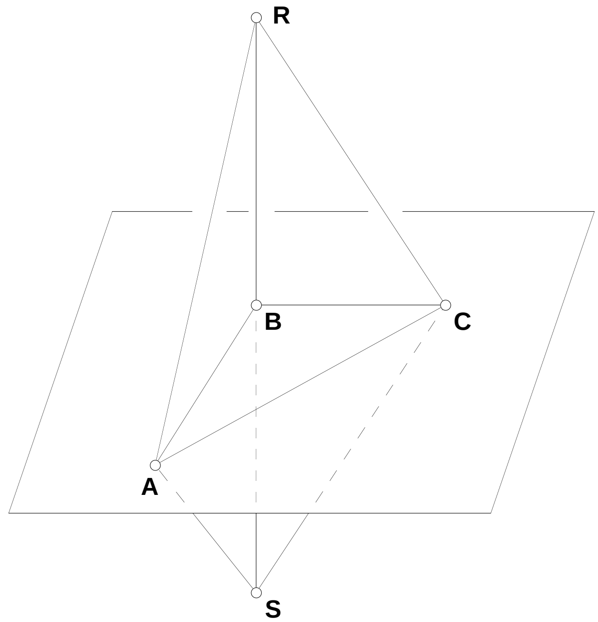

Figure 7