Math 460: Homework # 12.

1. (In this problem we prove the fact that you discovered in Problem 1 of Assignment 11.) See Figure 1. Given: I is the incenter of $\triangle ABC$ and the lines ID, IE and IF are perpendicular to AB, AC and BC respectively. To prove: the lines AF, BE and CD are concurrent. (Hint: use Theorem 35. And look up the definition of incenter.)

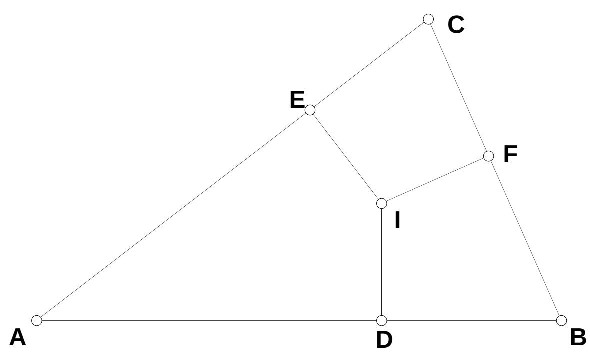

Figure 1

2. In Euclid's proof of Proposition 8, lines 13–17 use the method of "applying" one triangle to another. Find a way to replace lines 13–17 by an argument that doesn't use the idea of "applying." You may use anything that comes before Proposition 8, and you may also use Proposition 23 (since we have proved Proposition 23 without using Proposition 8.)

3. Now discover a different proof of Euclid Proposition 8 by contemplating Figure 3. You may use anything that comes before Proposition 6, and also Proposition 23. Do not use the method of "applying" one triangle to another, and do not use proof by contradiction. There are three cases, but you will get partial credit for doing one case.

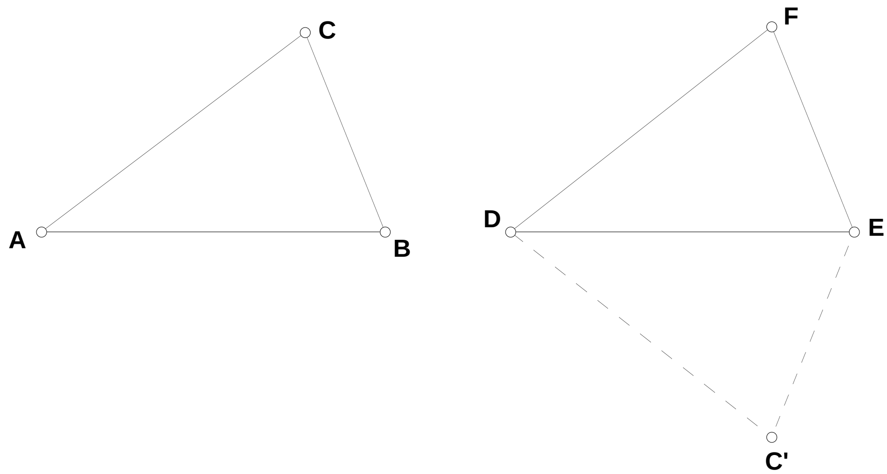

Figure 3

4. Prove the $\Longrightarrow$ direction of Theorem 41, using only the definition of tangent and facts from Euclid. (Hint: proof by contradiction; see Figure 4. If m is not perpendicular to OA, show that it is possible to construct a second point A' on m with OA=OA'.)

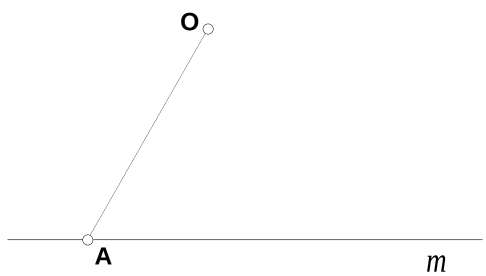

Figure 4

5. (See Figure 5.) Given: $\angle 1 = \angle 2$. To prove: $\angle ABC + \angle ADC = \angle BCD + \angle BAD$.

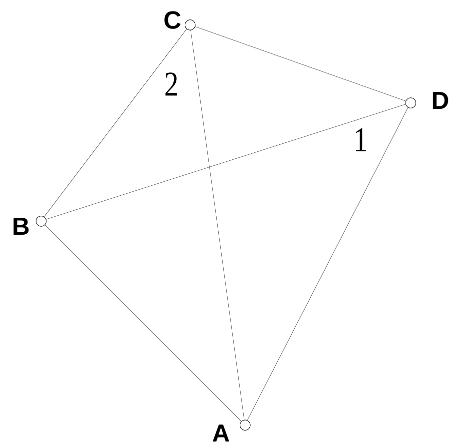

Figure 5

6. (See Figure 6.) Given: M is the midpoint of AB, and the lines that look concurrent are concurrent. To prove: DE is parallel to AB. (Hint: use Theorem 34 as one ingredient. Notice that if the product of the signed ratios is -1 then the product of the unsigned ratios is 1.)

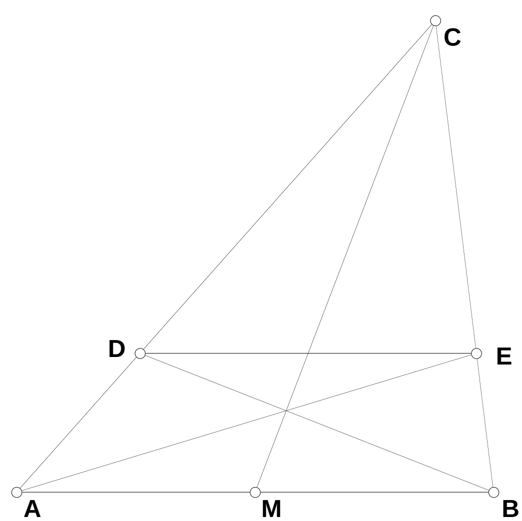

Figure 6

7. (See Figure 7.) Given: M and N are midpoints, $AC = DF$, $MC = NF$, $BC = EF$. To prove: $\triangle ABC \cong \triangle DEF$. (Hint: draw in one extra line in each triangle.)

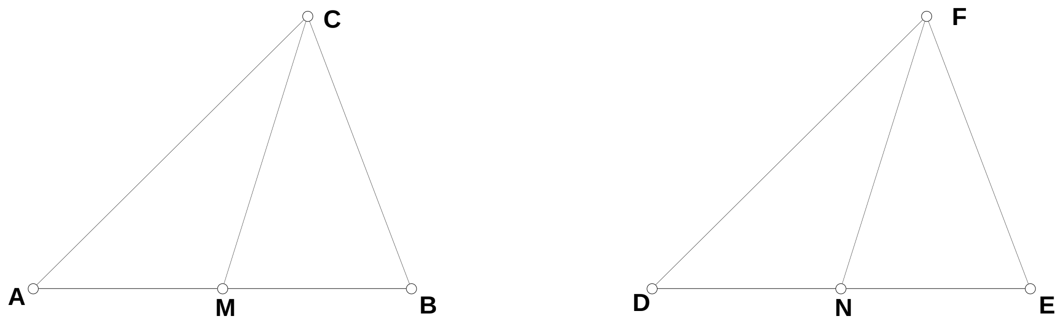

Figure 7

8. (See Figure 8.) Given: $\angle 1$ and $\angle 2$ are right angles. To prove: $\triangle ABC \sim \triangle AED$. (Hint: use Theorem 17 as one ingredient.)

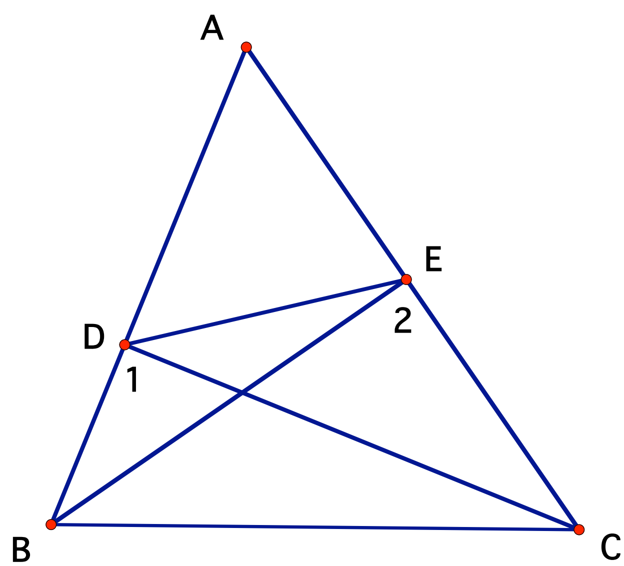

Figure 8