Math 460: Homework # 2.

This assignment covers up to Theorem 16 in the course notes (together with the definitions you need for Problem 1).

1. Read the definitions of perpendicular bisector (p. 28), circumcenter (p. 31), incenter (p. 32), altitude (p. 32), orthocenter (p. 32), median (p. 32) and centroid (p. 34). You don't have to do any other reading on these pages yet.

   Then use Geometer's Sketchpad to construct a triangle, along with the following

   (a) its circumcenter (labeled O)

   (b) its incenter (labeled I)

   (c) its orthocenter (labeled H)

   (d) its centroid (labeled G)

   (e) the line through O and H.

   Hide all the lines used in constructions (a)-(d). Print out a copy, then change the shape of the triangle and print another copy. The line through O and H has a special property that should be obvious from your pictures–what is it? (You do not need to prove anything for this problem.)

2. (Use Geometer's Sketchpad) Start with a triangle ABC. Let D and E be points on the segments AC and BC, respectively, with DE parallel to AB. Let F be the intersection of the segments DB and AE, and let G be the intersection of AB with the ray CF. What special property does G have? Display a measurement which shows it has this property. Print the picture, then change the shape of the triangle, check that G still has this property, and print the new picture. You do not have to prove anything for this problem.

3. (See Figure 1) Given $AB = AC = BC$ (that is, $\triangle ABC$ is an *equilateral* triangle). Let P be a point inside the triangle. Let a, b, and c be the distances from P to $\overleftrightarrow{AB}$, $\overleftrightarrow{AC}$ and $\overleftrightarrow{BC}$ respectively. Let h be the distance from A to $\overleftrightarrow{BC}$. To prove: $a + b + c = h$. (Hint: think about areas.)

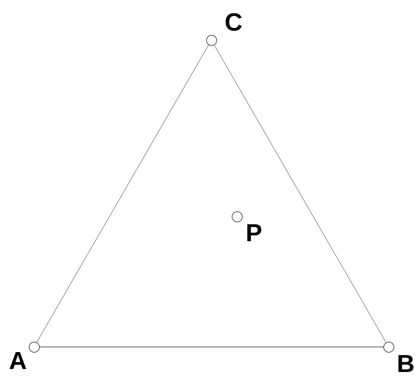

Figure 1

4. Given a quadrilateral $ABCD$ with $AB = BC$ and $CD = AD$, prove that the diagonals AC and BD are perpendicular.

5. (See Figure 2) Given: $ABCD$ is a parallelogram, E is the midpoint of CD, and F is the midpoint of BC. To prove: $DP = PQ = QB$. You do not need to draw any extra lines.

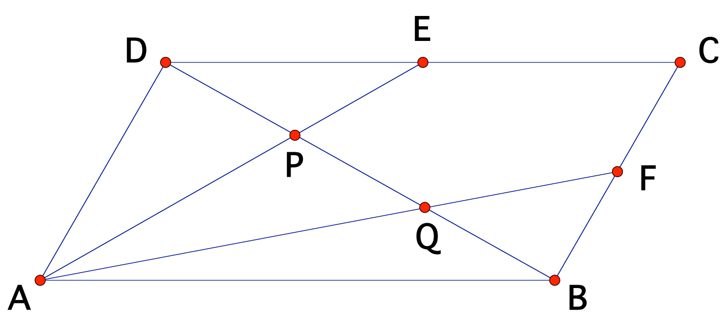

Figure 2

6. (See Figure 3) Given $MK = MQ, \angle K = \angle Q$, PM is perpendicular to MK, and LM is perpendicular to MQ, prove $RS = TS$. (Hint: Use what was shown in problem 5 of the first assignment.) **Warning:** Although it is true that equals subtracted from equals give equals, the same idea is *not* valid for congruence (it is not always true that congruent triangles subtracted from congruent triangles give congruent triangles.)

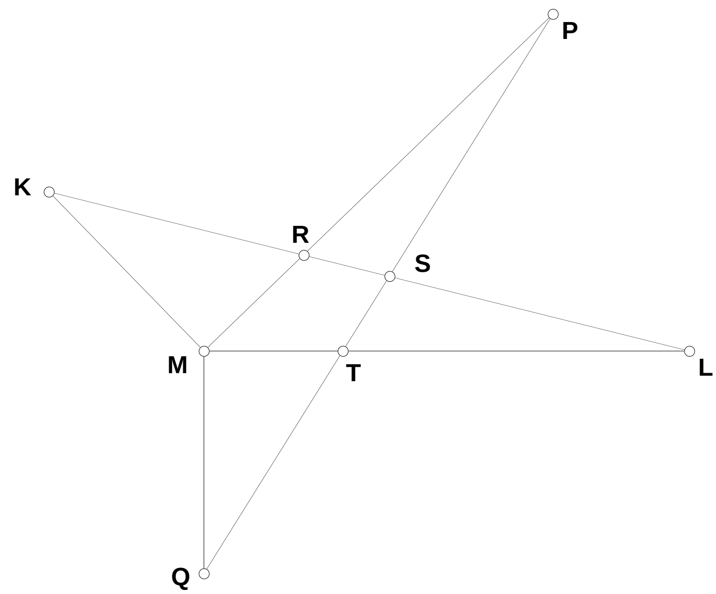

Figure 3

7. Let $ABCD$ be a quadrilateral, and let M, N, P, and Q be the midpoints of the sides. Prove that $MNPQ$ is a parallelogram.

8. (See Figure 4) Given: DE is parallel to AB, EF is parallel to BC, and DF is parallel to AC. To prove: $\triangle ABC$ is similar to $\triangle DEF$.

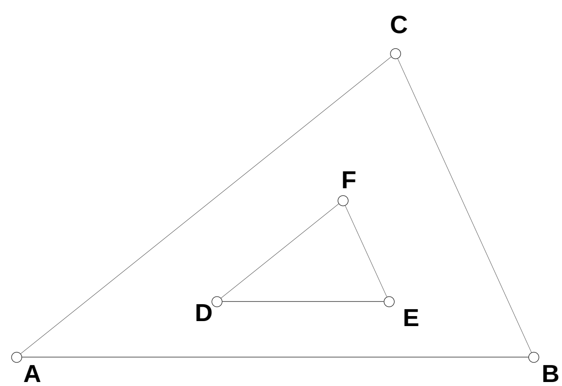

Figure 4

9. Let ABC be a triangle and let D and E be points on the segments AC and BC, respectively, with DE parallel to AB. Let M be the midpoint of AB, and let N be the intersection of DE and CM. Prove that N is the midpoint of DE. (Hint: Use two pairs of similar triangles).

10. (See Figure 5.) For this problem you need the definition of circle: a *circle* consists of all of the points which are at a given distance (called the *radius*) from a given point (called the *center*).

    Given: O is the center of the circle. To prove: $\angle AOC = 2\angle ABC$. (Hint: Use algebra as one ingredient in your proof.)

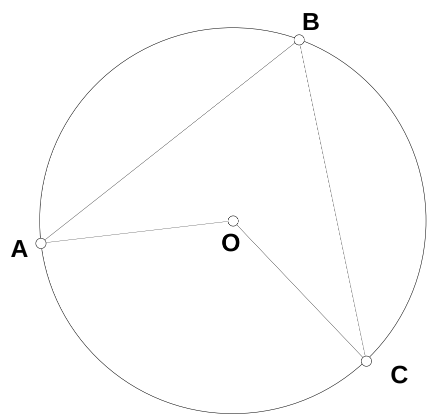

Figure 5