Math 460: Homework # 5.

This assignment covers up to the end of Section 5.1.

1. (Use Geometer's Sketchpad.) Draw a triangle ABC. Then draw a line ℓ through A parallel to BC, a line m through B parallel to AC, and a line n through C parallel to AB. Let D be the intersection of ℓ and m, E the intersection of ℓ and n, and F the intersection of m and n. Next, draw in the three altitudes of $\triangle ABC$ and the three perpendicular bisectors of $\triangle DEF$. What do you notice?

2. (Use Geometer's Sketchpad.) To say that a quadrilateral is *inscribed* in a circle means that all four of its vertices lie on the circle. Not every quadrilateral can be inscribed in a circle; when a quadrilateral can be inscribed in a circle its angles satisfy a certain equation. Find this equation and print out a copy of the picture with the calculations which demonstrates that the equation holds. You do not have to prove anything for this problem. (Note: for *every* quadrilateral it is true that the sum of the angles is $360°$, so this isn't the equation you're looking for.)

3. (In this problem we prove a fact that you demonstrated experimentally in Problem 1 of the fourth assignment.) Let $ABCD$ be a quadrilateral. Let M, N, P, and Q be the midpoints of the sides. Prove the area of $MNPQ$ is one half the area of $ABCD$.

4. (In this problem we prove the fact that you demonstrated experimentally in Problem 2 of the fourth assignment.) Let ABC be a triangle, and let P be a point in the interior of ABC. Construct the lines connecting P to each of the vertices, and let A', B' and C' be the points where these lines meet the sides BC, AC, and AB, respectively. Prove that
$$\frac{A'B}{A'C}\frac{B'C}{B'A}\frac{C'A}{C'B} = 1.$$
(Hint: Use Theorem 28 twice.)

5. (10 points) One of these three statements is very hard to prove. Prove the other two.

   (i) A triangle is isosceles $\Longleftrightarrow$ it has two equal altitudes.

   (ii) A triangle is isosceles $\Longleftrightarrow$ it has two equal angle bisectors.

   (iii) A triangle is isosceles $\Longleftrightarrow$ it has two equal medians.

   Note: since we have defined altitudes, angle bisectors, and medians to be lines, not line segments, the statements require some explanation. In the first statement, "altitude" means the part of the altitude that goes from the vertex to the opposite side, and similarly for the other two statements.

6. (See Figure 1). Prove Case (ii) of Theorem 28. Given: A', B' and C' are collinear. To prove:

$$\frac{A'B}{A'C}\frac{B'C}{B'A}\frac{C'A}{C'B} = 1.$$

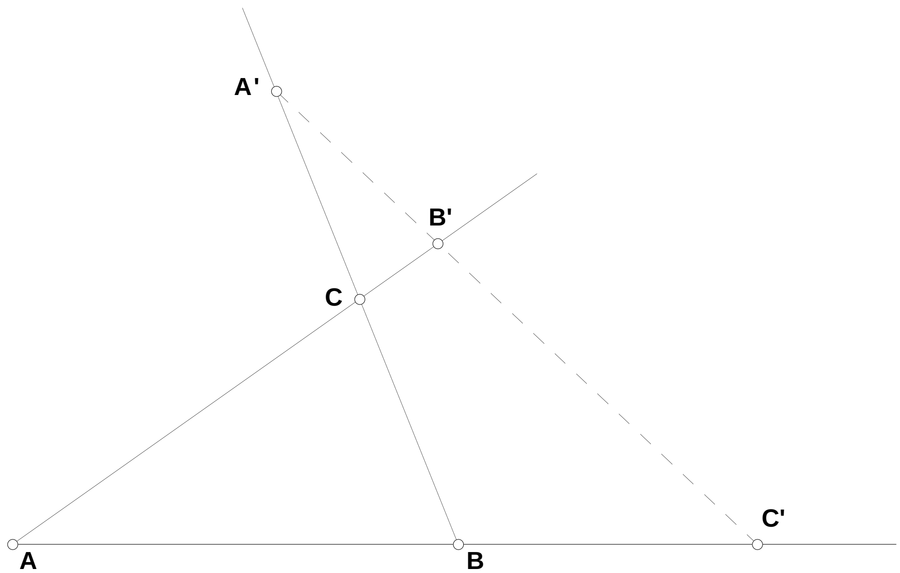

Figure 1

7. (See Figure 2.) Given: L is the midmoint of BC, M is the midpoint of AB, K is the midpoint of AF, and $BE \perp AC$. To prove: $\angle KML$ is a right angle.

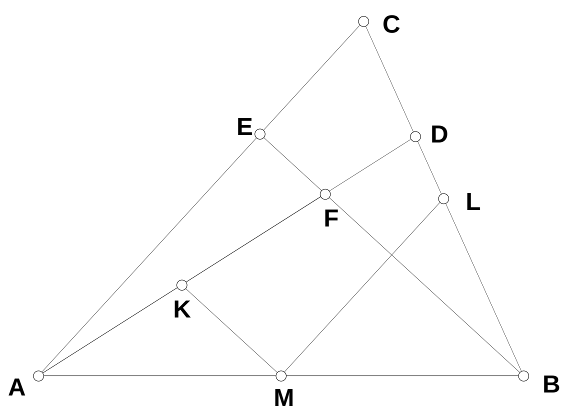

Figure 2

8. (See Figure 3.) Given: $\angle A = \angle B$, $AD = BE$, $\angle ADG = \angle BEF$. To prove: $\angle CFE = \angle CGD$.

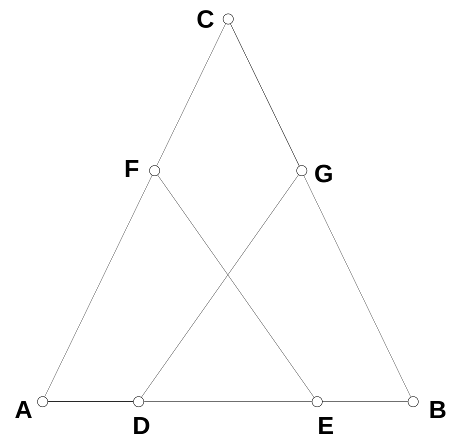

Figure 3

9. (See Figure 4.) Given: $ABCD$ is a parallelogram, M is the midpoint of BC and N is the midpoint of AD. To prove: $AQ = QP = PC$.

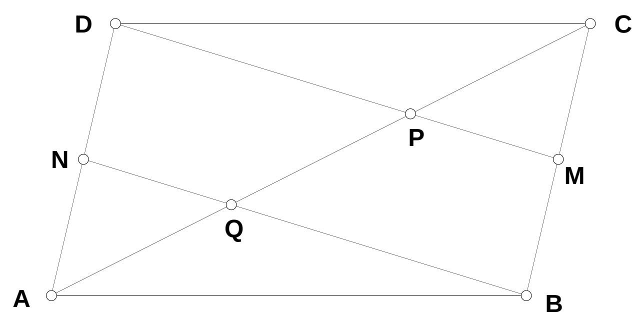

Figure 4