Math 460: Homework # 8.

1. (Use Geometer's Sketchpad.) Make a custom tool which constructs the orthocenter of a given triangle (your custom tool should create only the orthocenter, the altitudes should be hidden). Make a triangle ABC and use your custom tool to find the orthocenter. Label the orthocenter H. Then use your custom tool a second time, this time to find the orthocenter of triangle ABH. Indicate on your picture exactly where the orthocenter of triangle ABH is.

2. (Use Geometer's Sketchpad). Make a custom tool that carries out the construction described in the proof of Euclid Proposition 10, using only the "Circle by center and radius" and "Segment" commands (*not* the "Angle bisector" command). Print out a copy of this custom tool. (I'll show you in class how to print custom tools).

3. (In this problem we complete the proof of the fact that you discovered in Problem 1 of Assignment 6). See Figure 1. Given: M, N and P are midpoints; $DE = AP$, $DF = BN$, and $EF = CM$. To prove: the area of $\triangle DEF$ is 3/4 of the area of $\triangle ABC$. (Hint: The triangle $\triangle DEF$ turns out to be congruent to a triangle that appears in the Figures illustrating Problem 7 of Assignment 6 and Problem 3 of Assignment 7. You may use what you proved in those problems.)

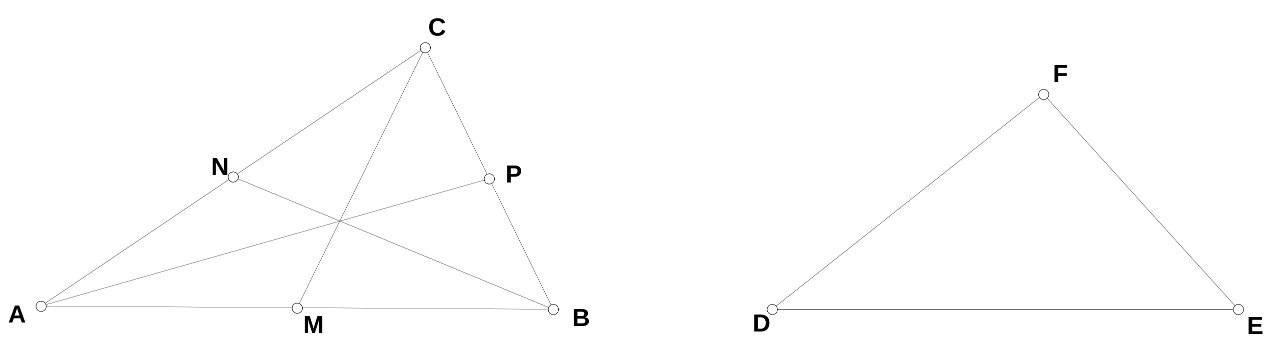

Figure 1

4. (In this problem we continue to prove what you discovered in Problem 2 of Assignment 6.) See Figure 2. Given: G is the centroid of $\triangle ABC$, M, N and P are the midpoints of AB, AC and BC respectively, and U, V, W, X, Y and Z are the centroids of the six "little triangles." To prove: the lines UV, XW and GB are concurrent. (Hint: Use a proof similar to that of Theorem 26. You will find it helpful to consider the dashed lines in the picture. You may use Problem 4 of Assignment 7.)

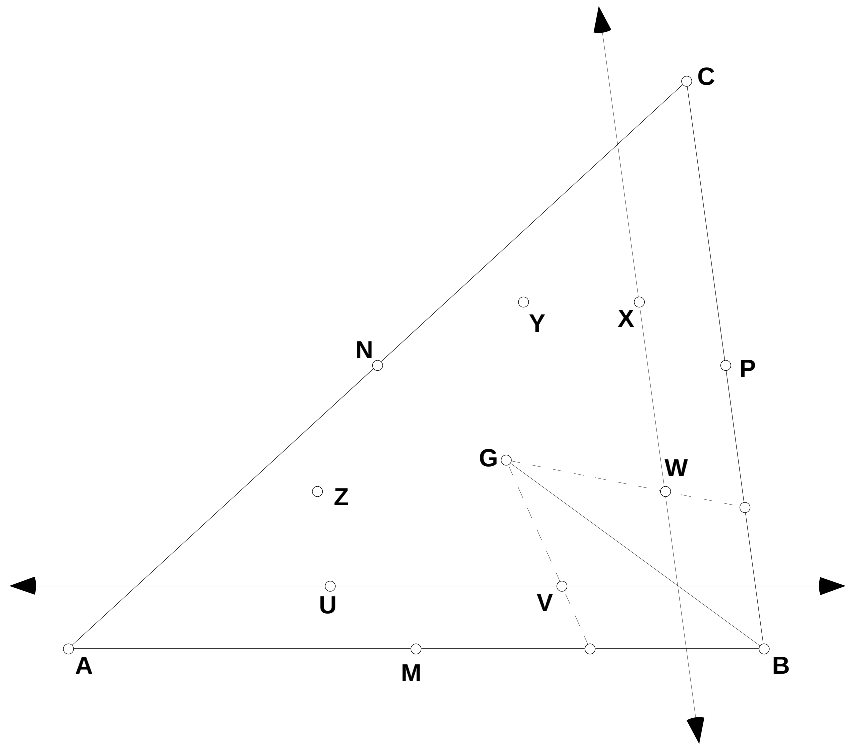

Figure 2

5. (See Figure 3.) Given: M, N and P are the midpoints of AB, AC and BC. To prove: $\dfrac{AD}{DB} = \dfrac{1}{2}$.

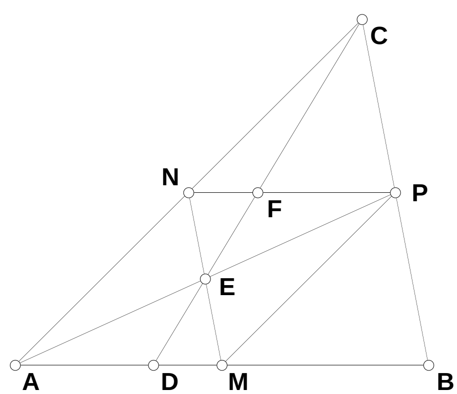

Figure 3

6. (In this problem we prove what you discovered in Problem 2 of Assignment 7.) See

Figure 4. Given: the things that look like squares are squares. To prove: the areas of the shaded triangles are all equal.

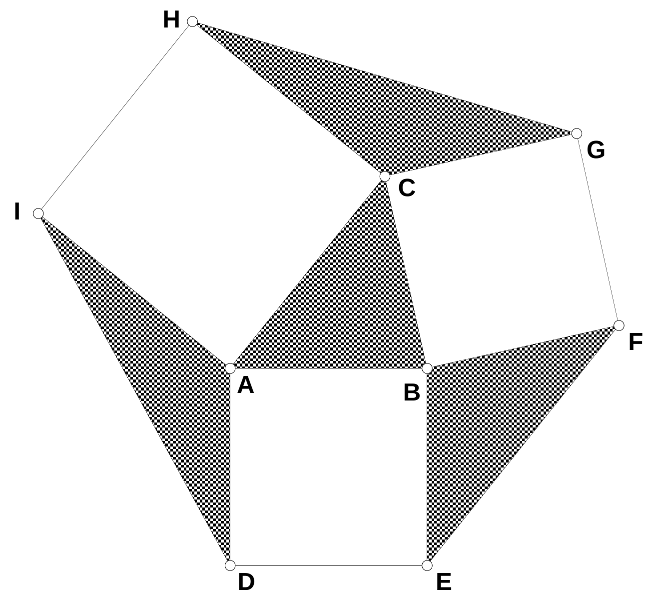

Figure 4

7. (See Figure 5.) Given: $AC = BC$ and $AD = BF$. To prove: $DE = EF$. Do not draw in any extra lines!

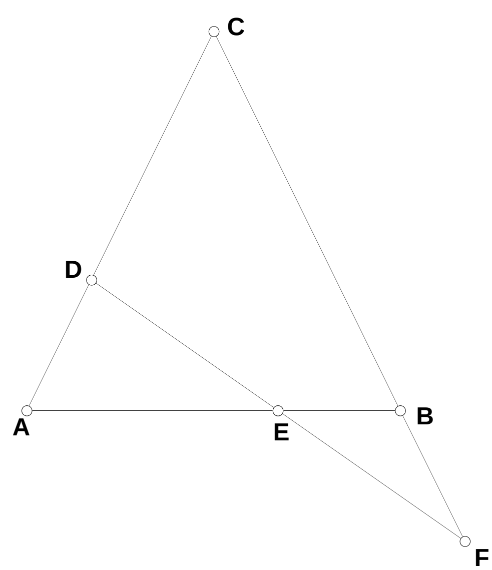

Figure 5

8. Use Theorem 35 and Problem 8 from Assignment 7 to give a new proof of Theorem 23.

9. (10 points)

   (a) Given a segment AB and a point C (which may or may not be on AB). Prove:

   $$C \text{ is on the perpendicular bisector of } AB \iff AC = BC.$$

   (b) Use part (a) to give a shorter proof of Theorem 22. Begin as usual by constructing two perpendicular bisectors and their intersection X. Then connect X to the three vertices. After that you may not add anything to the picture and you may not use any Basic Fact or Theorem; you may only use part (a).