1. Circle the letters corresponding to statements which are true for any two square matrices A and B such that $AB = 0$:

A. Either $A = 0$ or $B = 0$.

B. $\mathcal{C}(A) \subset \mathcal{N}(B)$.

C. $\mathcal{C}(A^T) \supset \mathcal{N}(B^T)$.

D. $BA = 0$.

E. Either $A = 0$ or B is singular.

2. Which of the following is a subspace of the vector space $C[0,1]$ of continuous functions on $[0,1]$? Circle the letters corresponding to correct answers.

A. Functions with $f(1) = 0$.

B. Functions with $f(0) = 1$.

C. Functions with $\int_0^1 f(x)\,dx = 0$.

D. Functions with $f(0) = f(1)$.

E. Functions with $f(0) \le f(1)$.

3. Which of the following is a linear transformation T of the space of 3×3 matrices? Circle the letters corresponding to correct answers.

A. $T(A) = U$ where U is the row echelon form of A.

B. $T(A) = A^{-1}$.

C. $T(A) = A^T$.

D. $T(A) = P$ where $p = Pb$ is the orthogonal projection of b to $\mathcal{C}(A)$.

E. $T(A) = E_{2,1}(3)A$ where $E_{2,1}(3)$ is an elementary matrix.

4. Let $A = \begin{pmatrix} 1 & 2 & 2 \\ -6 & -11 & -12 \\ 1 & -4 & 3 \end{pmatrix}$.

a) Factor $A = LU$.

b) Find A^{-1}.

5. Let T be the transformation of the space $\mathbf{P}_2$ of polynomials of degree 2 defined as $T(p(x)) = p(x) - p(1)$.

a) Write the matrix of T in the basis 1, x, x^2 of $\mathbf{P}_2$.

b) Find the rank of T.

c) Find the fundamental subspaces $\mathcal{C}(T)$ and $\mathcal{N}(T)$.

Let $\mathbf{v_1} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, $\mathbf{v_2} = \begin{bmatrix} 4 \\ 0 \\ 0 \end{bmatrix}$, $\mathbf{v_3} = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$, $\mathbf{v_4} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$, $\mathbf{v_5} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$.

6. With $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$, $\mathbf{v}_4$, $\mathbf{v}_5$ as above, which one of the following is a spanning set for R^3?

A. $\mathbf{v_1}$, $\mathbf{v_2}$, $\mathbf{v_3}$

B. $\mathbf{v_2}$, $\mathbf{v_3}$, $\mathbf{v_5}$

C. $\mathbf{v_1}$, $\mathbf{v_5}$

D. $\mathbf{v_1}$, $\mathbf{v_2}$, $\mathbf{v_3}$, $\mathbf{v_4}$

E. $\mathbf{v_1}$, $\mathbf{v_2}$, $\mathbf{v_3}$, $\mathbf{v_5}$

7. With $\mathbf{v}_1, \mathbf{v}_2$, $\mathbf{v}_3$, $\mathbf{v}_4$, $\mathbf{v}_5$ above, which set is linearly independent?

A. $\mathbf{v_1}$, $\mathbf{v_2}$, $\mathbf{v_3}$

B. $\mathbf{v_2}$, $\mathbf{v_3}$, $\mathbf{v_5}$

C. $\mathbf{v_1}$, $\mathbf{v_5}$

D. $\mathbf{v_1}$, $\mathbf{v_2}$, $\mathbf{v_3}$, $\mathbf{v_4}$

E. $\mathbf{v_1}$, $\mathbf{v_2}$, $\mathbf{v_3}$, $\mathbf{v_5}$

8. With $\mathbf{v_1}$, $\mathbf{v_2}$, $\mathbf{v_3}$, $\mathbf{v_4}$, $\mathbf{v_5}$ as above, which set is a basis for R^3?

A. $\mathbf{v_1}$, $\mathbf{v_2}$, $\mathbf{v_3}$

B. $\mathbf{v_2}$, $\mathbf{v_3}$, $\mathbf{v_4}$

C. $\mathbf{v_1}$, $\mathbf{v_5}$

D. $\mathbf{v_1}$, $\mathbf{v_2}$, $\mathbf{v_3}$, $\mathbf{v_4}$

E. $\mathbf{v_1}$, $\mathbf{v_2}$, $\mathbf{v_3}$, $\mathbf{v_5}$

9. Which of the following are subspaces of R^3.

(1) The set of all (x, y, z) such that $xy = 0$.

(2) The set of all (x, y, z) such that $y = 2x$ and $z = 3x$.

(3) The set of all (x, y, z) such that $x + y = 7z + 1$

(4) The set of all (x, y, z) such that $x^2 + y^2 - z^2 = 0$

(5) The set of all (x, y, z) such that $x + y + z = 4$

A. (2)
B. (1) and (2)
C. (3) and (5)
D. (2), (3) and (5)
E. (1), (2) and (4)

10. Determine which matrix A represents the linear transformation $T : \mathcal{P}_3 \to \mathcal{P}_3$ defined by $T(p) = x\dfrac{dp}{dx}$, and with the ordered basis $\{1, x, x^2, x^3\}$ for $\mathcal{P}_3$:

A. $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

B. $\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \end{pmatrix}$

C. $\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

D. $\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

E. $\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{3} \\ 0 & 0 & 0 & 0 \end{pmatrix}.$

11. Let $a = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$, $b = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$.

i) Find the projection matrix that projects onto the line through a.

ii) Find the point on the line through a which is closest to b.

iii) Find the projection of b onto the column space of $A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{pmatrix}$.

12. Let $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 6 & 8 & 10 & 12 \end{bmatrix}$.

i) Find a basis for the null space of A.

ii) Find a basis for the row space of A.

iii) Find a basis for the column space of A.