Review Problems for the Final Exam/MA 213, Fall 2013

$\big($Exam time: Monday, December 16, 10:30 am-12:30 pm, CB 106$\big)$

The final exam is comprehensive and covers Chapters 12.1-12.6, 13.1-13.5, 14.1-14.7, 15.1-15.6, 16.1-16.4, and Chapter 17.1-17.2. The review problem sheets for the two midterm exams and the following review problem for Chapters 16.1-16.4, 17.1-17.2 can be viewed as the whole review set for the final exam.

- Review the definition formula for calculating curl $\mathbf{F}$, where $\mathbf{F}$ is a vector field in $\mathbf{R}^2$ or $\mathbf{R}^3$.
- Review the statements for Green's Theorem, Stokes' Theorem.
- Review parametric equations for surface, and the formula for its normal vector, and surface area elements.
- Review the definition formula for line integrals, surface areas, and surface integrals.

Review problems from Chapter 16 and Chapter 17

1. Evaluate $\int_C x\sin y\,dx + xyz\,dz$, where C is given by $x=t, y=t^2, z=t^3$ for $0\le t\le 1$.

2. Evaluate $\int_C x^3y\,dx - x\,dy$, where C is the circle $\{(x,y)|x^2+y^2=1\}$ with counterclockwise orientation.

3. Evalulate $\int_C \mathbf{F}\cdot d\mathbf{s}$, where $\mathbf{F}(x,y)=(x^2y, e^y)$, $C=\{(t^2,-t^3)|0\le t\le 1\}$.

4. Use the Green's theorem to evaluate $\int_C x^2y\,dx - xy^2\,dy$, where C is $\{(x,y)|x^2+y^2=4\}$ with counterclockwise orientation.

5. Use the Green's theorem to evaluate $\int_C \mathbf{F}\cdot d\mathbf{s}$, where

$$\mathbf{F}(x,y)=\Big\langle y-\ln(x^2+y^2),\ 2\tan^{-1}(\frac{y}{x})\Big\rangle$$

and C is the circle $(x-2)^2+(y-3)^2=1$ oriented counterclockwise.

6. Calculate curl of the vector fields

$$\mathbf{F}(x, y, z) = \frac{1}{\sqrt{x^2 + y^2 + z^2}}(x, y, z),$$

and

$$\mathbf{G}(x, y, z) = (y^2z^3, 2xyz^3, 3xy^2z^2).$$

7. Determine whether or not the vector field is conservative. if it is conservative, find a function f such that $\mathbf{F} = \nabla f$.

$$\mathbf{F}(x, y, z) = \Big\langle y\cos(xy), x\cos(xy), -\sin z\Big\rangle,$$

and

$$\mathbf{F}(x, y, z) = \Big\langle 2xy, (x^2 + 2yz), y^2\Big\rangle.$$

8. Calculate the area of the surface

$$z = \frac{2}{3}(x^{\frac{3}{2}} + y^{\frac{3}{2}}),\ 0 \le x \le 1,\ 0 \le y \le 1$$

9. Calculate the area of the surface with parametric equations $x = u^2,\ y = uv,\ z = \frac{1}{2}v^2,\ 0 \le u \le 1,\ 0 \le v \le 2$.

10. Calculate $\int\int_S \mathbf{F} \cdot d\mathbf{S}$, where $\mathbf{F}(x, y, z) = \langle xy, yz, zx\rangle$ and S is the part of the paraboloid $z = 4-x^2-y^2$ that lies above the square $0 \le x \le 1, 0 \le y \le 1$, and has upward orientation.

11. Use Stokes' Theorem to evaluate $\int_C \mathbf{F} \cdot d\mathbf{s}$, where $\mathbf{F}(x, y, z) = \langle x+y^2, y+z^2, z+x^2\rangle$ and C is the triangle with vertices $(1, 0, 0)$, $(0, 1, 0)$, and $(0, 0, 1)$.

12. Use Stokes' Theorem to evaluate $\int\int_S \mathrm{curl}\mathbf{F} \cdot d\mathbf{S}$, where $\mathbf{F}(x, y, z) = (2y\cos z, e^x\sin z, xe^y)$ and S is the hemisphere $x^2 + y^2 + z^2 = 9,\ z \ge 0$, oriented upward.