Review Problems for Midterm II (MA 266-066 & 067, Fall 20156)

The second midterm exam will be taken place at

8:00-9:00 pm, Wednesday, November 9, 2016, MATH175

Sections to be covered

The sections to be covered in this example contain

1. Chapter 3.3: Complex roots
2. Chapter 3.4: Repeated real roots
3. Chapter 3.5: Method of undetermined coefficients
4. Chapter 3.6: Variation of parameters
5. Chapter 3.7: Mechanical vibrations
6. Chapter 3.8: Forced vibrations
7. Chapter 4.1-Chapter 4.3: Higher order constant coefficient ODEs
   (Methods of characteristics and undetermined coefficients)
8. Chapter 6.1: Laplace transform
9. Chapter 6.2: Solution of Initial Value Problems

Some Review Problems

1. Solve the initial value problem

$$y'' + 2y' + 2y = 0,\ y(\frac{\pi}{4}) = 2,\ y'(\frac{\pi}{4}) = -2.$$

2. Consider the initial value problem

$$y'' + 2ay' + (a^2 + 1)y = 0,\ y(0) = 1,\ y'(0) = 0.$$

(a) Find the solution $y(t)$ of this problem, and
(b) for $a = 1$ find the smallest T such that $|y(t)| < 0.1$ for $t > T$.
3. Solve the initial value problem

$$9y'' + 6y' + 82y = 0,\ y(0) = -1,\ y'(0) = 2.$$

4. Consider the initial value problem

$$9y'' + 12y' + 4y = 0,\ y(0) = a,\ y'(0) = -1.$$

(a) Solve the initial value problem, and
(b) find the critical value of a that separates solutions that become negative from those that are always positive.
5. Use the variation of parameters to find the second solution of

$$(x - 1)y'' - xy' + y = 0,\ x > 1;\ y_1(x) = e^x.$$

6. Solve the initial value problem by the method of undetermined coefficients

$$y'' + 2y' + 5y = 4e^{-t}\cos 2t,\ y(0) = 1,\ y'(0) = 0.$$

7. Find the correct form of a particular solution of

$$y'' + 4y = t^2 \sin 2t + (6t + 7)\cos 2t.$$

8. Use the variation of parameters to find the general solution of

$$x^2y'' - 3xy' + 4y = x^2 \ln x,\ x > 0;\ y_1(x) = x^2,\ y_2(x) = x^2 \ln x.$$

9. A spring is stretched 10 cm by a force of 3 N. A mass of 2kg is hung from the spring and is also attached to a viscous damper that exerts a force of 3 N when the velocity of the mass is $5m/s$. If the mass is pulled down 5cm below its equilibrium position and given an initial downward velocity of $10cm/s$, determine its position u at any time t.
10. Find the general solutions of

$$y^{(4)} + y = 0,$$

$$y^{(4)} - 5y^{(2)} + 4y = 0.$$

11. Find the general solution of

$$y^{(4)} + 6y''' + 17y'' + 22y' + 14y = 0.$$

12. Solve the initial value problem of

$$y''' - 3y'' + 2y' = t + e^t,\ y(0) = 1,\ y'(0) = -\frac{1}{4},\ y''(0) = -\frac{3}{2}.$$

13. Find a suitable form of particular solution of

$$y^{(4)} + 2y''' + 2y'' = 3e^t + 2te^{-t} + e^{-t}\sin t.$$

14. Use the definition to find the Laplace transform of the following functions:
(a) $f(t) = t^2$, $t > 0$.
(b)

$$f(t) = \begin{cases} t, & 0 \le t < 1 \\ 1, & 1 \le t < +\infty. \end{cases}$$

15. Use Laplace transform to solve the initial value problem:

$$y'' + \omega^2 y = \cos 2t,\ \omega^2 \neq 4,\ y(0) = 1,\ y'(0) = 0.$$

$$y^{(4)} - 4y''' + 6y'' - 4y' = y = 0;\quad y(0) = 0,\ y'(0) = 1,\ y''(0) = 0,\ y'''(0) = 1.$$