Set of review problems of Midterm I/MA266, Fall 2017

The first midterm exam takes place

October 4 2017, Wednesday, 8:00-9:00 pm/MATH 175

It covers

Chapter 2.1–Chapter 2.7, & Chapter 3.1–Chapter 3.3

There will be six problems. Here are some sample problems that you can practice. The topics that you need to review include

1. How to solve first order linear ODE?
2. How to solve first order separable ODE?
3. How to check a first order ODE is exact? If yes, how to solve it? How to find the integrating factors in the two simple case if the equation is not exact, then solve it?
4. How to solve second order constant coefficient linear ODEs when the characteristic equation has two distinct real roots or two conjugate complex roots?
5. How to determine the types (e.g., stable, unstable, or semi-stable) of equilibrium solutions of autonomous equation and logistic models?
6. How to implement the first few steps of Euler's method for numerical solutions to ODEs.
7. Modeling by differential equations, such as the water tank problem and population growth/decay models.

(1) Solve the initial value problem of the following differential equations:

$$y' + \frac{2}{t}y = \frac{1+\cos t}{t^2}; \quad y(\frac{\pi}{2}) = 1.$$

$$(1+t^2)y' + 4ty = (1+t^2)^{-2}; \; y(1) = 2.$$

(2) First find a solution to the initial value problem of the differential equation and then specify the domain of solution:

$$\frac{dy}{dx} = \frac{1+3x^2}{3y^2-6y}, \; y(0) = 1.$$

$$(x^2 + 3xy + y^2)\,dx - x^2\,dy = 0; y(1) = 1.$$

(3) Consider the differential equation:

$$\frac{dy}{dt} = (y-3)(9-y).$$

First determine the stable and unstable equilibrium solutions. Then find general solutions of the equation. (Hint: use the partial fraction method $\frac{1}{y(9-y)} = \frac{A}{y-3} + \frac{B}{9-y}$)).

(4) Apply the phase diagram analysis to determine the equilibrium points and classify each one asymptotically stable, unstable, or semistable.

$$\frac{dy}{dt} = y^2(16-y^2).$$

(5) First verify that the differential equation

$$(e^x \sin y - 2y\sin x + x^2)\,dx + (e^x\cos y + 2\cos x + y^2)\,dy = 0$$

is exact. Then find a general solution of the differential equation.

(6) Find the value of b for which the given equation is exact:

$$(ye^{2xy} + x) + bxe^{2xy}y' = 0.$$

(7) A $200m^3$ tank is full of water that contains a pollutant Z at a concentration of $0.5g/m^3$. Cleaner water, with a pollutant concentration of $0.2g/m^3$ is pumped into the well-mixed tank at a rate of $5m^3/sec$. Water flows out of the tank through an overflow valve at the rate of $6m^3/sec$. Determine the amount and concentration of pollutant in the tank as a function of the time elapsed since cleaned water is pumped.

(8) Consider the initial value problem

$$y' = t^2 + y^2;\ y(0) = 1.$$

Use Euler's method with $h = 0.1$ to approximate $y(1.3)$.

(9) Find a solution to the differential equations:

$$y'' + 5y' + 3y = 0;\ y(0) = 1,\ y'(0) = -2.$$

$$y'' - 2y' + 5y = 0;\ y(\frac{\pi}{2}) = 1,\ y'(\frac{\pi}{2}) = -1.$$

(10) If the differential equation $t^2y'' - 2y' + (3+t)y = 0$ has y_1 and y_2 as a fundamental set of solutions and if $W(y_1, y_2)(2) = 3$, find the value of $W(y_1, y_2)(4)$.